Probabilistic fatigue life prediction of multidirectional composite laminates

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Abstract

A new damage accumulation model for fatigue life prediction of composite laminates is proposed in this study. The model is constructed on the ply level and uses a new multiaxial damage index to consider the damage caused by different stress components. The fatigue life is predicted based on the proposed model and experimental results of the unidirectional laminates. The numerical results are compared with the experimental data for balanced laminates. The predicted fatigue lives agree with the experimental observations very well. The methodology for probabilistic fatigue life prediction and reliability calculation is also presented.

Keywords: Composite; Multidirectional laminates; Fatigue; Damage; Probabilistic

1. Introduction

Composite materials are widely used in automotive, naval, and aerospace structures, where they are often subjected to cyclic fatigue loading [1]. Fatigue is one of the most common failure modes in all structural materials, including composite materials. The development of microstructures for advanced high-performance composites has mainly focused on achieving high modulus and high strength. However, along with high strength, such materials must also be able to absorb energy and resist cyclic fatigue loads. Fatigue life prediction of composite material enables fatigue-resistant design and inspection and maintenance decision-making.

Due to the anisotropic properties of composite materials, the fatigue problem is multiaxial. There is extensive progress in multiaxial fatigue analysis of metals, but much further effort is needed for composite materials [2–5]. Accurate fatigue life prediction needs to include the random variation in material properties (Young’s modulus, strength, etc.), loading history, component geometry, and environmental conditions.

This paper proposes a simple and versatile damage accumulation model for multiaxial fatigue problem in laminated composites. The model is extended to progressive failure of multidirectional laminates. The model parameters are obtained through analysis of test data on several unidirectional laminates. The numerical simulation results for several balanced laminates are compared with the experimental results. The material properties and geometry properties are randomized based on the experimental data or some assumptions. Monte Carlo simulation is used to calculate the distribution of fatigue life under different load amplitudes. The reliability of one type of multidirectional laminates under different load levels is computed and compared.

2. Existing fatigue models

Fatigue analysis of composite materials is difficult due to several basic characteristics of the composite material [6]. However, many attempts have been made for fatigue modeling and life prediction of fiber-reinforce polymers. Degrieck and Van Paepegem [7] classify existing fatigue models into three categories: fatigue life model ($S$–$N$ curves), residual strength or residual stiffness model, and progressive damage model.

The fatigue life model is established based on $S$–$N$ curves or Goodman diagrams. This approach does not consider the details of the damage mechanism. It is entirely empirical and needs a lot of experimental data. For every variation in laminates (different stacking...
sequence and ply orientation), a new set of specimens are needed to develop the $S$–$N$ curves, thus making this approach expensive and time-consuming. But this methodology is easy to apply and a lot of commercial software packages are available for use. The failure criteria mimic the form of static strength criteria, based on two major failure modes (fiber failure and matrix failure) [8,9].

\[
\sigma_1 = \sigma_{1u}^i
\]

\[
\left( \frac{\sigma_2}{\sigma_{2u}^i} \right)^2 + \left( \frac{\tau}{\tau_u} \right)^2 = 1
\]

where $\sigma_1$ and $\sigma_2$ are the stresses along the fiber direction and transverse to the fiber direction respectively, and $\tau$ is the shear stress. $\sigma_{1u}^i$, $\sigma_{2u}^i$, and $\tau_u$ are the ultimate strengths of the three stress components. They are functions of the stress level, stress ratio and the number of stress cycles. The relationship is expressed in $S$–$N$ curves from previous experimental data.

Wu [10], Jen and Lee [11] proposed different failure criteria based on the Tsai–Hill criterion. Philippidis and Vassilopoulos [12] proposed a failure criterion based on the Tsai–Wu criterion. All these methods use the fatigue strength (corresponding to a given $N$, from the $S$–$N$ curves) instead of the ultimate strength in the Tsai–Hill or Tsai–Wu criteria.

Other researchers directly use the family of $S$–$N$ curves to calculate the fatigue life [13–15]. The family of $S$–$N$ curves includes stress ratio, load frequency and other factors affecting the shape of the $S$–$N$ curves. The main objective is to use the same computing methodology to account for different loading conditions.

The progressive model seems to be more accurate because it accounts for the detailed failure mechanism of the composite material. But for accurate analysis, this model requires that the damage introduced by local failure be correlated with the material properties degradation. A quantitative relationship in this regard is difficult to find and needs extensive experimental data. Also this model is computationally expensive and complicated, and thus is difficult to apply directly to engineering design.

The residual strength and the stiffness model consider damage accumulation. The idea of the residual strength model is simple and easy to apply. But the damage evolution function is assumed and calibrated through constant amplitude tests. For composite materials, the damage mechanism is different under different stress levels and also depends on the load sequence. It is hard to use a simple damage accumulation rule to describe the damage evolution under complicated loading conditions.

The residual strength and stiffness model are based on damage mechanics, which relates fatigue failure to the damage evolution process. The degradation of stiffness or strength is correlated with a damage variable (damage index). Different damage evolution functions [16–22,30] have been suggested based on some assumptions or experimental results. The failure is assumed to occur when the cumulative damage reaches a critical value (usually unity). The general form of the damage accumulation rule is:

\[
\frac{dD}{dN} = f(D, \sigma_i, \varepsilon_i, N, \ldots)
\]

where $D$ is the damage index, $\sigma_i$, $\varepsilon_i$ are the stress and strain components, and $N$ is the number of load cycles. The parameters in the damage model are calibrated through experimental observations or through reasonable assumptions.

Unlike the above two approaches in fatigue analysis, which are at the macroscopic level, the progressive failure model considers local damage mechanisms, such as delamination, local ply buckling, fiber breakage, etc. All these local damage mechanisms lead to damage accumulation to the macroscopic material. Global failure occurs once the damage introduced by the local failure exceeds the global allowable level. This method is computationally complicated because it accounts for many failure mechanisms and is also related to damage accumulation. Tserpes et al. [23] gives a progressive damage model which includes seven local failure modes, including material stiffness degradation.

The fatigue life model is easy to use and has an experimental data base. Also the commercial software and methodology are available to calculate the fatigue life. However, this model requires a lot of experimental work, which is sometime cost prohibitive. Most of the fatigue life models do not consider damage accumulation and are difficult to extend to complicated loading condition.

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3. Numerical model of fatigue life prediction

A new and simple model based on damage mechanics is suggested here. It is basically an $S$–$N$ curve-based fatigue progressive damage model and ignores the detailed analysis of the local failure. It uses fatigue data from the family of $S$–$N$ curves and uses a special damage variable to account for the multiaxial fatigue in each ply. A detailed description is given below.

For general fatigue calculation using $S$–$N$ curves, first a set of experiments is conducted considering different stress levels and specimen geometries. Then the fatigue life and applied stress level are plotted together. A curve fitting method is used to find the relation between the fatigue life and stress level. The empirical relationship is then used to predict the fatigue life of the real structure [24]. In this study, the maximum cyclic stress is used to
The general formula is:

\[
S_{\text{max}} = A_R \times \log(N) + B_R
\]

\[
R = \frac{S_{\text{max}}}{S_{\text{min}}}
\]

where \(S_{\text{max}}\) and \(S_{\text{min}}\) are the maximum and minimum cyclic stresses, \(R\) is the stress ratio, \(N\) is the fatigue life, and \(A_R\) and \(B_R\) are the strength coefficients corresponding to the stress ratios. Actually, Eq. (3) is a family of \(S-N\) curves, which are expressed in a semi-log manner.

The above description is for one-dimensional constant amplitude loading. The damage concept is needed for multidimensional complicated loading conditions. In damage mechanics, damage evolution is expressed through the material property degradation process. When the damage reaches the unity, the whole material is assumed to fail. The damage increases monotonically as the loading history increases. Under cyclic fatigue loading, the damage is usually expressed as the fraction of the number of failure cycles. A linear damage accumulation function, Miner’s rule, is popularly used:

\[
D = \sum_{i=1}^{K} D_i = \sum_{i=1}^{K} \frac{n_i}{N_i} = 1
\]

where \(K\) is the number of variable loading stages, \(D_i\) is the damage caused in each loading stage, \(n_i\) is the number of cycles in the \(i\)th loading stage, and \(N_i\) is the constant amplitude fatigue life estimated from Eq. (3).

For multidirectional composite laminates, each ply is generally under multiaxial stress. One way to calculate the fatigue life is to perform the fatigue experiment under different stress states to get the \(S-N\) curves. Due to the large number of possible combination of each stress components, this method is impractical. Therefore, a new damage index is suggested based on the Tsai–Hill static strength failure criterion, for which only the experiment for unidirectional laminates is required and the fatigue life for a general stacking sequence and ply orientation can be predicted based on the model. The Tsai–Hill criterion is for a single ply in the form of Eq. (5) [25]

\[
\frac{\sigma_1}{F_1} + \frac{\sigma_2}{F_2} + \frac{\tau_6}{F_6} - \sigma_1 \sigma_2 = 1
\]

where \(\sigma_1, \sigma_2\) are the stresses along the fiber direction and transverse to the fiber direction respectively, \(\tau_6\) is the shear stress, and \(F_1, F_2, F_6\) are the static strengths of different directions.

The damage in a single ply caused in one cycle under stress state \((\sigma_1, \sigma_2, \tau_6)\) is assumed to have the form:

\[
D_i = \frac{1}{\sqrt{\left(\frac{N_i^1}\cdot \frac{N_i^2}\cdot \frac{N_i^3}\cdot \frac{N_i^4}\cdot \frac{N_i^5}\cdot \frac{N_i^6}\right)}}
\]

where \(N_i^1, N_i^2, N_i^6\) are the fatigue lives estimated for unidirectional laminates under pure stress components \(\sigma_1, \sigma_2, \text{ and } \tau_6\) respectively. The sign for the normal stress interactive term is chosen as positive in considering the monotonically increasing damage. As shown in Eq. (6), the fatigue experiments are only needed along the longitudinal direction, along the transverse direction and under shear loading for unidirectional laminates.

Using the new damage index, the Miner’s rule is rewritten as:

\[
D = \sum_{i=1}^{K} D_i = \sum_{i=1}^{K} \frac{n_i}{N_i} = 1
\]

Eq. (7) is the proposed general form for multiaxial fatigue damage accumulation. The ply is assumed to fail if the accumulated damage index exceeds unity.

The above discussion is easily extended to a laminate with multiple plies, along the following steps. Divide the total loading history into several blocks. In each block, check the failure of each ply using Eq. (7). If no failure occurs, accumulate the fatigue damage for each ply caused in this block and progress to the next step. If failure occurs, assume that the ply strength and stiffness decrease to zero. Then update the global stiffness matrix and progress to the next step. The computation is continued till the entire laminate fails. The number of loading cycles to failure is the fatigue life of the composite laminate. The flow chart of the computational procedure is shown in Fig. 1.

The above discussion was limited to deterministic considerations. Due to the high randomness inherent in

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Fig. 1. Flow chart of the computational procedure.
the composite fatigue problem [29], it is more appropriate to use a probabilistic approach to evaluate the fatigue life and reliability of the structure.

The methodology shown above can be easily extended to probabilistic analysis. First the material properties are treated as random variables and are randomized based on the scatter in the experimental data or on appropriate assumptions. The geometric properties of the laminate are also treated as random variables, say, thickness of the laminate, orientation of the laminate, etc. Monte Carlo simulation is used to calculate the fatigue life distribution. Under different load levels, a certain number of Monte Carlo samples are used to calculate the fatigue lives. The fatigue lives are plotted and fitted to a probabilistic distribution. Once the fatigue life distribution is obtained, it is easy to calculate the reliability at the different number of loading cycles.

4. Validation of the proposed model

Fatigue properties of a glass–fiber-based composite, available from the US Department of Energy/Montana State University (DOE/MSU) Composite Materials Fatigue Database [26], are used in this section to validate the proposed fatigue calculation methodology and to develop a general method to take into account the randomness of the fatigue problem.

The material chosen, D155, has fatigue test data for both unidirectional and multidirectional laminates. The static material properties are listed in Table 1.

<table>
<thead>
<tr>
<th>Volume fraction $V_f$ (%)</th>
<th>Elastic modulus (GPa)</th>
<th>Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_1$</td>
<td>$E_2$</td>
</tr>
<tr>
<td>45</td>
<td>35.0</td>
<td>8.99</td>
</tr>
</tbody>
</table>

The geometry properties and volume fraction of balanced laminates using D155 are shown in Table 2. A balanced laminate consists of pairs of layers with identical thickness and elastic properties but with $+\theta$ and $-\theta$ orientations [25].

The volume fraction ($V_f$) is not uniform across the ply orientation specimens. An empirical formula has been suggested for including the effect of volume fraction on the elastic properties (Eq. (8)) [26].

$$
\begin{align*}
E_1 &= \left( \frac{1}{32.71} \right) (3.1 + 65.8V_f) \\
E_2 &= \left( \frac{1}{2.206} \right) (1 + 0.836V_f) \\
G_{12} &= \left( \frac{1}{2.809} \right) (1 + 1.672V_f) \\
v_{12} &= \left( \frac{1}{0.318} \right) (0.35 - 0.15V_f)
\end{align*}
$$

where $V_f$ is volume fraction of the fiber, $E_1$, $E_2$, $G_{12}$, $v_{12}$ are the elastic properties shown in Table 1, and $E_{1t}$, $E_{2t}$, $G_{12t}$, $v_{12t}$ are the elastic properties of volume fraction $V_f$.

For the unidirectional fatigue experiment along the fiber direction, several groups of specimen were tested with different volume fractions. The test results for stress ratio 0.1 are shown in Fig. 2. Fig. 3 shows the semi-log $S$–$N$ plot with stress level $S_{max}$ normalized by the volume fraction $V_f$, whereas in Fig. 2 the stress level is not normalized. As shown in the plot, the normalized data has less scatter. Therefore the regression model with normalized data is used for the $S$–$N$ curve. During the calculation, it will be first transformed to the actual $S$–$N$ curve.

<table>
<thead>
<tr>
<th>Material number</th>
<th>Angle (degree)</th>
<th>Volume fraction $V_f$ (%)</th>
<th>Mean value of thickness (mm)</th>
<th>Std. dev. of thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D155-0</td>
<td>0\degree</td>
<td>39.9</td>
<td>2.70</td>
<td>0.11</td>
</tr>
<tr>
<td>D155-10</td>
<td>±10\degree</td>
<td>35.5</td>
<td>3.47</td>
<td>0.17</td>
</tr>
<tr>
<td>D155-20</td>
<td>±20\degree</td>
<td>38.4</td>
<td>3.21</td>
<td>0.14</td>
</tr>
<tr>
<td>D155-30</td>
<td>±30\degree</td>
<td>39.6</td>
<td>3.11</td>
<td>0.14</td>
</tr>
<tr>
<td>D155-40</td>
<td>±40\degree</td>
<td>38.9</td>
<td>3.17</td>
<td>0.09</td>
</tr>
<tr>
<td>D155-45</td>
<td>±45\degree</td>
<td>38.9</td>
<td>3.17</td>
<td>0.06</td>
</tr>
<tr>
<td>D155-50</td>
<td>±50\degree</td>
<td>38.1</td>
<td>3.23</td>
<td>0.11</td>
</tr>
<tr>
<td>D155-60</td>
<td>±60\degree</td>
<td>39.6</td>
<td>3.11</td>
<td>0.14</td>
</tr>
<tr>
<td>D155-70</td>
<td>±70\degree</td>
<td>38.9</td>
<td>3.17</td>
<td>0.04</td>
</tr>
<tr>
<td>D155-80</td>
<td>±80\degree</td>
<td>37.1</td>
<td>3.32</td>
<td>0.10</td>
</tr>
<tr>
<td>D155-90</td>
<td>±90\degree</td>
<td>37.1</td>
<td>3.32</td>
<td>0.12</td>
</tr>
</tbody>
</table>
curve according to the specific volume fraction of the specimen.

The fatigue tests were also performed in the transverse direction. Similar to the longitudinal direction, the un-normalized and normalized results for stress ratio 0.1 are plotted in Figs. 4 and 5.

In the present fatigue model, the $S-N$ curve is also required for pure shear fatigue of unidirectional laminates. However, this type of data is not available in the data base, perhaps due to the difficulty in performing pure shear test on unidirectional laminates. A simplified estimation is drawn as follows. Analysis of fatigue test data on many types of materials and specimens in the literature [27] has shown that the slope for $S-N$ curves fall in a range from $-0.14$ to $-0.07$. In this study, the average value of longitudinal and transverse slopes is taken as the slope of the shear fatigue $S-N$ curve. Degallaix et al. [28] studied monotonic and fatigue shear tests on one type of unidirectional glass-epoxy composite. They observed that the $y$-intercept of the pure shear fatigue $S-N$ curve is 49.9 MPa. The static shear strength of the material is 64 MPa. The ratio of the $y$-intercept and static strength is 0.78. In this section, this ratio value is calibrated by using test data from a balanced laminate ($\frac{1}{2}/C_{6}^{45}/C_{138}^{90}$), which shows that a value of 0.7 is appropriate for the current material (D155). The un-normalized and normalized $S-N$ curve formulas for in-plane shear fatigue are then found to be:

$$S_{\text{max}} = -5.8524 \times \log(N) + 59.729$$

$$S_{\text{max}}^\ast = \frac{S_{\text{max}}}{V_f} = -13.005 \times \log(N) + 132.73 \quad (9)$$

After determining all the material properties for the unidirectional laminate, the fatigue life of the laminate with an arbitrary stacking sequence and ply orientation can be predicted using the methodology in Section 3. The numerical prediction results and experimental results for balanced laminates are shown in Figs. 6–16. All the numerical results use five randomly generated
Fig. 7. Comparison of numerical prediction and experimental data for \[\{\pm 10\}_3\] laminates.

Fig. 11. Comparison of numerical prediction and experimental data for \[\{\pm 45\}_3\] laminates.

Fig. 8. Comparison of numerical prediction and experimental data for \[\{\pm 20\}_3\] laminates.

Fig. 12. Comparison of numerical prediction and experimental data for \[\{\pm 50\}_3\] laminates.

Fig. 9. Comparison of numerical prediction and experimental data for \[\{\pm 30\}_3\] laminates.

Fig. 13. Comparison of numerical prediction and experimental data for \[\{\pm 60\}_3\] laminates.

Fig. 10. Comparison of numerical prediction and experimental data for \[\{\pm 40\}_3\] laminates.

Fig. 14. Comparison of numerical prediction and experimental data for \[\{\pm 70\}_3\] laminates.
samples for each stress level. The detailed statistics of the random samples are shown in the next section.

From Figs. 6–16, it is seen that the numerical prediction results agree very well with the experimental data, thus validating the proposed method.

5. Fatigue life distribution and reliability estimation

A balanced angle laminate \( (\pm 45^\circ) \) is used here to illustrate the reliability estimation method. The statistics of the input random variables are listed in Table 3.

Table 3

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Mean value</th>
<th>Standard deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_f )</td>
<td>0.389</td>
<td>0.05 ( 0.389 )</td>
<td>Normal</td>
</tr>
<tr>
<td>Thickness ( t )</td>
<td>3.17 mm</td>
<td>0.06 mm</td>
<td>Normal</td>
</tr>
<tr>
<td>Angle</td>
<td>45(^\circ)</td>
<td>3(^\circ)</td>
<td>Normal</td>
</tr>
<tr>
<td>( E_1 )</td>
<td>30.7 GPa</td>
<td>0.05 ( 30.7 ) GPa</td>
<td>Normal</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>8.0 GPa</td>
<td>0.05 ( 8.0 ) GPa</td>
<td>Normal</td>
</tr>
<tr>
<td>( V_{12} )</td>
<td>0.376</td>
<td>0.05 ( 0.376 )</td>
<td>Normal</td>
</tr>
<tr>
<td>( G_{12} )</td>
<td>3.57 GPa</td>
<td>0.05 ( 3.57 ) GPa</td>
<td>Normal</td>
</tr>
<tr>
<td>All fatigue S–N curve coefficients</td>
<td>As shown in the experimental results from Figs. 3–6</td>
<td>0.05 ( \text{mean value} )</td>
<td>Normal</td>
</tr>
</tbody>
</table>

Fig. 15. Comparison of numerical prediction and experimental data for \( [\pm 80^\circ]_3 \) laminates.

Fig. 16. Comparison of numerical prediction and experimental data for \( [\pm 90^\circ]_3 \) laminates.

Fig. 17. Histogram of the numerical life predictions and normal distribution fit under different applied stress levels.
Five thousand Monte Carlo simulations are performed under four different applied stress levels and the fatigue life is computed using the proposed method in Section 3. The histogram of the fatigue life is plotted and fitted to a normal distribution using commercial software MINITAB R14, as shown in Fig. 17. The empirical CDF of the numerical life prediction and the normal distribution fit are shown in Fig. 18. The reliability of the laminate is calculated and plotted in Fig. 19.

6. Conclusions and future work

A new multiaxial damage accumulation model for multidirectional composite laminates is suggested in this paper. The computational methodology for fatigue life prediction and reliability evaluation is also developed. Numerical simulation results under constant load amplitude are compared with available experimental data. The agreement is more than reasonable. Thus the proposed method shown in this paper appears both quantitatively and qualitatively appropriate for the probabilistic fatigue life prediction. Another advantage of the proposed model is that only the fatigue tests for unidirectional laminates are needed to predict the fatigue life of multidirectional laminates.

Although only constant amplitude fatigue life prediction is considered in this paper, the model has no difficulty in including variable amplitude loading. Future work is needed to validate the current method to variable amplitude loading.

The model was validated in Section 4 under a stress ratio of 0.1, for a tension–tension fatigue problem. The applicability of the current model needs further validation or modification for other stress ratios, and for tension–compression and compression–compression fatigue problems.

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References