ME3 Fundamentals of Fracture Mechanics

Lecture notes 2014-15

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Introduction

- **Aims**

  1. To develop an understanding of the various aspects involved in the area of fracture mechanics.
  2. To develop from first principles the basic ideas and equations needed for an understanding of fracture mechanics
  3. To define the advantages and disadvantages of this approach for studying the failure of materials and structures.
  4. To indicate how the basic principles may be applied to a range of industrial problems and materials.
  5. To lay foundations for the ME4 *Advanced Forming and Fracture* course.

- **Recommended background reading**

  1. TL Anderson, *Fracture Mechanics — Fundamentals and Applications* (3rd ed.). Taylor and Francis (2012),
  2. D Broek, *Elementary Fracture Mechanics*. Martinus Nijhoff (1987).
  3. AJ Kinloch and RJ Young, *Fracture Behavious of Polymers*. Elsevier (1983).
  4. JG Williams, *Fracture Mechanics of Polymers*. Ellis Horwood (1984).
  5. D Hull, *An Introduction to Composite Materials*. Cambridge University Press (1981).
  6. FL Matthews and RD Rawlings, *Composite Materials: Engineering and Science*. Chapman and Hall (1994).

RELATED LINKS

Link to PDF of TL Anderson, "Fracture Mechanics".

Contents

Nomenclature

Units conform to those adopted by the ESIS Task Group on Polymer Testing, 1988.

- ## Crack size and specimen geometry

Symbol	Meaning	Units
a	Crack length (or size)*1	m
Δa	Incremental crack length	m
B	Thickness	m
W	Second relevant dimension (often width)	m
A	Crack surface area	m^2
δA	Incremental crack surface area	m^2

*1 For a central notch the crack length is generally given as $2a$.

- ## Crack size and specimen geometry

Symbol	Meaning	Units
E	Tensile or compressive modulus*1	GPa
E'	Storage modulus	GPa
E''	Loss modulus	GPa
E^*	Complex modulus	GPa
μ	Shear modulus	GPa
C	Specimen compliance $C = u/P$	m/N
u	Displacement in load direction under load P	m
σ	Macroscopic stress (remote from crack tip)	MPa (N/mm^2)
ν	Lateral contraction ratio (Poisson's ratio*2)	—
$2r_p$	Crack tip plastic zone size	m
σ_y	Yield stress of material	MPa

*1 The term "Young's modulus" is limited to small scale linear elasticity.
*2 The term "Poisson's ratio" is limited to linear elasticity

- ## Crack size and specimen geometry

Symbol	Meaning	Units
$\sigma(r,\theta)$	Stress at arbitrary point (r,θ) referenced from crack tip	MPa
σ_{xx}	Stress in direction of crack propagation	MPa
σ_{yy}	Stress perpendicular to crack plane	MPa
σ_{zz}	Stress in crack plane normal to direction of crack propagation	MPa
σ_{xy}	Shear stresses	MPa

List of formulae

$$\sigma_{yy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right]$$

$$\sigma_{xx} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right]$$

$$K_1 = Y\sigma\sqrt{a}$$

$$\delta = \frac{\pi\sigma^2 a}{Ef}$$

$$\delta = \frac{8fa}{E\pi}\ln\left[\sec\frac{\pi\sigma}{2f}\right]$$

$$\delta_c = \frac{G_{1c}}{\sigma_y} = \frac{K_{1c}}{E'\sigma_y}$$

$$K_c^2 = EG_c = E\sigma_y\delta_c$$

$$K_{1c}^2 = \left(\frac{E}{1-\nu^2}\right)G_{1c}, K_{1c}^2 = EG_{1c} = mE\sigma_y\delta_c$$

$$B_{min} = 2.5\left[\frac{K_{1c}}{\sigma_y}\right]^2$$

$$a,\ (W-a), B \geq 2.5\left[\frac{K_{1c}}{\sigma_y}\right]^2$$

$$\sigma_f = \sqrt{\frac{EG_c}{\pi a}}$$

$$\sigma_f = \sqrt{\frac{EG_{1c}}{\pi a\left(1-\nu^2\right)}}$$

$$\sigma_f = \sqrt{\frac{4E\gamma_m}{\pi c}}$$

$$\sigma = \sigma_{yy} = \frac{K}{\sqrt{2\pi r}}$$

$$r_p = \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2$$

$$r_p = \frac{1}{6\pi}\left[\frac{K_{1c}}{\sigma_y}\right]^2$$

$$\bar{a}_{max} = \frac{\delta_c E\sigma_y}{2\pi\sigma_1^2} \text{ for } \frac{\sigma_1}{\sigma_y} < 0.5$$

$$\bar{a}_{max} = \frac{\delta_c E}{2\pi(\sigma_1 - 0.25\sigma_y)} \text{ for } \frac{\sigma_1}{\sigma_y} > 0.5$$

$$Y^2 a_{actual} = \bar{a}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$G_{1c} = \frac{P^2}{2B}\cdot\frac{\partial C}{\partial a}$$

$$Z = \frac{C}{\partial C/\partial(a/W)}$$

$$C = \frac{8a^3}{Bh^3E_{11}}$$

$$E_{11} = \frac{P}{u}\cdot\frac{8a^3}{Bh^3}$$

Chapter 1 Modes of failure

Getting fracture in perspective, as one amongst several distinct failure modes for engineering structures.

Engineering design methodology requires that the designer should be aware of the possible modes of failure of a component or structure, so that the design process can be carried out with a view to ensuring the avoidance of all possible, relevant, failure modes. In some respects, one of the major skills in designing is being able to correctly identify the most probable failure mechanism. Almost all the classic failure stories from industry relate to machines or objects where the designer got it wrong, sometimes with tragic consequences.

A classification of the more common failure modes known for structural components can be made as follows:

- Failure by elastic instability (*buckling*);
- Failure by excessively large elastic deformations (*jamming*);
- Failure by gross plastic deformation (*yielding*);
- Failure by tensile instability (*necking*);
- Failure by fast fracture (*cracking*, *snapping*);
- Failure by environmental corrosion (*rusting*, *rotting*).

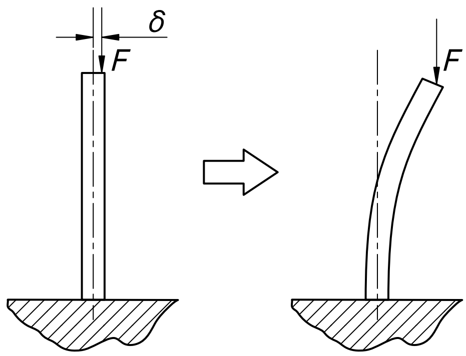
This course aims to demonstrate and explain the techniques available for 'designing against fracture'. However, a brief study of each one of the failure modes listed above is given for the following purposes:

1. To put fracture mechanics into context for the design engineer, and
2. To provide some background information on how the development of materials which exhibit good performance in terms of resistance to failure by yielding, effectively encouraged a relatively new type of failure: by cracking.

1.1 Buckling and jamming

Buckling is typically a risk for long, slender members in compression.

The phenomenon of buckling originates from small misalignments in the application of the load when the elastic restoring forces in the slender member are no longer sufficient to keep the system in equilibrium. This condition usually results in instability with catastrophic deformations until the bent column yields or fractures, or its ends touch:



Jamming can occur when, as a result of an oversight in design, excessively large elastic deflections take place. It is a risk in the design of engines, for example, when clearances between components are very small.

Avoidance of both types of failure can be ensured by geometric specifications.

Currently much research is being carried out on the development of high modulus materials, often containing fibres, to allow high stresses to be applied without the development of high strain values. The reality is that in practice, the available ratios of Young's Modulus to density (E/ρ) do not offer the design engineer a broad spectrum from which to choose.

1.2 Yielding

The engineer understands this term to mean both localised yielding and failure by plastic collapse.

A failure by yielding can occur with general yielding or with the onset of limited plastic deformation in the component in question.

From Knott *Fundamentals of Fracture Mechanics*:

"A body is said to have undergone general yielding when it is no longer possible to trace a path, across the load bearing section, through elastically deformed material only."

In the past, design would invariably aim to avoid the onset of any yielding. Current design methods can use localised yielding or plastic collapse as the limiting criteria in a certain design situations.

Plastic collapse can be used as a safety feature in emergency situations, for example, in the choice of Armco crash barriers for use as the central reservation of a motorway or around race tracks: large plastic deformation of the barrier is desirable so that the large forces experienced in an accident can be absorbed with less risk to the drivers.

Plastic deformation can also be desired and induced in certain situations in order to create beneficial residual stresses or to blunt sharp defects.

Examples:

- autofrettaging of tubes;

- proof testing of pressure vessels beyond yielding.

In the design against failure by plastic collapse, the engineer is no longer restricted to a range of geometries or a limited choice of elastic constants. A wide choice of materials with various yield strengths is available.

RELATED LINKS

Example of Armco barrier installation

1.3 Necking

A risk for tension members subjected to a soft (load-controlled) loading.

Necking can only happen as a result of a gross overload and depends on the interaction of material properties with the structure's geometry and the applied stress system.

Assuming that problems with buckling/jamming and necking can be prevented by design of the structural member and by limiting tensile stresses then the failure mode to guard against is yielding.

In order to design against necking failures, design codes have been developed and the application of safety factors ensures that necking failure is highly unlikely. However, the economic imperative of the last fifty years has led to attempts to use higher stresses for a given geometrical configuration requiring materials of higher uniaxial strength. The development of these high strength materials and their efficient usage has rendered structures prone to failure by an alternative mode of failure: namely fast fracture or cracking.

1.4 Cracking

Progressive separation of a structure into two pieces by the creation of new surface area.

Fast fracture is the unstable propagation of a crack in a structure and is almost invariably produced by applied stresses apparently less than the design stress calculated with the appropriate design code. The resulting catastrophic nature of these failures led to the development of **Fracture Mechanics**. These failures were often described by the term brittle, applied in the macro sense rather than as a description of the micromechanisms of crack extension.

A **brittle fracture** is one in which the onset of unstable crack propagation is produced by an applied stress less than the general yield stress of the uncracked ligament remaining when instability first occurs.

These failures are usually associated with gross stress concentrations in large components or structures and with loading systems which don't relax the applied stresses as the crack extends. Although in steels these fractures happen at low temperatures and/or in thick sections, for both aluminium and steel they can also take place in very thin sheets.

Chapter 2 Origins of fracture mechanics

An account of how and why fracture mechanics emerged as a distinct discipline.

Why do materials fracture ?

To try to answer this question we need to start by answering the more fundamental question for engineers and society at large — why should we be concerned about fracture ?

The answer is hopefully obvious, and intuitively it seems that society was always concerned with fracture.

2.1 Historical

Previously exploited to shape hard, strong, natural materials, fracture later became a problem for more ductile materials.

In the Stone Age man used a variety of materials as well as stone. Some of the first craftsmen to engage in series production of useful items were the flint axehead makers and they appreciated that flint was a hard, relatively strong but under some circumstances hopelessly brittle material. Flint axeheads could be shaped by fracturing flint or other stones to give the rough outline shape required and this was normally accomplished by judicious hammering against another stone to cause the axehead to fracture along planes of weakness.

Useful though they were for crushing the heads of the odd sabre tooth tiger the flint axehead has a nasty tendency to shatter when struck against the ground or rocks — not a very useful property !

Bronze age man improved things no end by developing a material capable of being moulded to virtually any form and possessing the wondrous property of being ductile. However bronze is, if anything, too soft to make good cutting tools or weapons.

With the dawn of the Iron Age and ensuing centuries the artisans who were the precursors of the modern day engineer really had a material which possessed a good balance of strength and hardness but still a material with an annoying propensity to fracture unexpectedly.

Since approximately the beginning of this century engineers plus the odd metallurgist and physicist have been trying to answer the question “how do I stop it breaking or fracturing?”

The first approach was one still in common usage today. Because nobody really appreciated the mechanism by which materials fractured the approach taken was to overdesign the component by accepting the brittle nature of the material of construction and limiting the stress on the component to some small fraction of the tensile strength. At the same time it became commonplace to proof test structures and components by subjecting them to a much larger load or stress than they would see in service.

Early cannons and muskets were proof tested by inserting an extra large charge (double or treble charge of gunpowder) and firing the piece. If the gun survived in one piece then the chances were very good that it would survive in service for a reasonable period of time. The bascules of Tower Bridge were proof tested by parking horse drawn carts filled with large iron weights all over the bridge decking until it was deemed that the load was double that which might be experienced in reality. The recent centenary of Tower Bridge would indicate that this design philosophy has some merit. The bill for the construction of Tower Bridge on the other hand demonstrates the inadequacy of this approach. Safety factors approaching 10 on yield or tensile strength do not make for a cheap construction! Despite this design philosophy and the generally low applied stresses utilised, catastrophic fractures continued to occur from time to time in a wide variety of components and structures. Steam boilers and railway equipment were particularly troublesome!

2.2 Liberty ships

A single, notorious case which motivated the modern study of fracture.

It was not until the 1940s when a series of catastrophic failures of steel structures gave sufficient impetus that attention was turned to attempting to answer the far more fundamental questions of "why and how does it break?".

As part of the wartime Lend-Lease agreement between the US and UK it became obvious that the UK did not have enough commercial shipping capacity to be able to transport the quantities of materiel required from the US to UK ports. Additionally one of our European neighbours was deliberately fracturing our ships faster than we could build them. The US government therefore called for tenders to build a large number of general purpose cargo ships and tankers with the express purpose of transporting weapons, food and oil from the Eastern seaboard of the US to the UK. The tender required that these vessels should be built in a matter of a few months rather the years required by conventional riveted plate construction. The majority of N. American shipyards said it was impossible but a Californian civil engineer, curiously enough called Kaiser, claimed that he could meet the deadlines using a novel construction method of a ship assembly line and all welded construction. The history of these so-called 'Liberty' ships is well known. Suffice it to say that of the ~2500 ships built, over 140 broke in two and nearly 700 suffered serious cracking problems, some when lying in port but invariably in cold weather.

At the end of the second world war a commission was set up to try to answer the question as to why these ships had failed. Tests on plates from the fractured ships showed that in order not to fail by catastrophic cleavage fracture the ships plate had to have a minimum value of Charpy Energy of about 35 J at 0°C and exhibit less than 70% crystallinity. It was further determined that all the serious cracking was by brittle fracture from either preexistent flaws or stress concentrations in steel plates which did not meet this criterion.

2.3 Griffith and Irwin

Two 20th century researchers on whose work the modern subject of Fracture Mechanics is constructed.

A far more fundamental piece of research had already been carried out by A.A. Griffiths, a British physicist who, in 1920, had addressed the problem of why glass fibres fracture at stress levels approximately two orders of magnitude below their theoretical strength. Griffiths recognised that the separation of glass is a fracture dominated process in which fracture is inevitable if the extension of an existing crack lowers the overall energy of the system. This apparently very simple concept is an example of an energy balance (thermodynamic) approach to fracture in which the decrease in elastic strain energy of the cracked body is counteracted by the energy needed or required to create the two new crack surfaces.

The major advance on this earlier theory was due to G.R. Irwin, who in the late 1940s pointed out that to apply a Griffith criterion to the fracture of metallic materials required that instead of considering the energy balance as being between the strain energy of the body and the surface energy term, as is the case for a truly brittle material like glass, the energy balance for a metallic material should be between the elastic strain energy and the surface energy plus the work done in plastic deformation. Most importantly Irwin also recognised that for a metallic material the work done in producing the plastic deformation is invariably orders of magnitude greater than the surface energy term.

Thus the basis for fracture mechanics came about with the definition of a material property G which is defined as the total energy absorbed during a unit increment of crack length per unit thickness. Nowadays G is invariably referred to as the **strain energy release rate**.

Only a few years later Irwin made a further fundamental step by showing that it was possible to reconcile the concept of a critical stress intensity causing fracture, K_c, with the idea of a critical value of the strain energy release rate, G_c. The realisation that the strain energy and stress intensity approaches to the prediction of fracture are equivalent led to a rapid development in the discipline of Linear Elastic Fracture Mechanics (LEFM) which allows engineers to predict what defects are tolerable in a given structure under known loading conditions — the basic goal of Fracture Mechanics.

2.4 Fracture mechanics today

The analysis and prediction of fracture steadily improve, but materials are loaded ever closer to their strength limits.

Curiously enough, as our knowledge of fracture mechanics has improved the number of catastrophic fracture incidents has continued to increase in absolute terms. However, we are becoming very good at explaining the cause of such catastrophic fractures, albeit after the event. A major reason for this is quite simply the increased usage of high performance materials (HS steels, HS Al alloys, titanium alloys, ceramics etc.) which are more fracture susceptible as we will see later. A secondary reason may well be due to the fact that the increasing complexity of modern structures and machines renders exhaustive analysis of all possible loading configurations very difficult and time consuming.

This apparent lack of success should not be taken as an indictment of the inadequacy of our understanding of the causes of fracture events. Brittle fracture is sometimes potentiated by financial imperative which drives manufacturers to try to improve margins by "extending" the operating service envelope of components to the extent that intrinsic safety factors become compromised in the extreme event. Too often when a major component or structure fails in such a catastrophic manner it becomes apparent during the post mortem design review that the original component or structure was not designed using any form of fracture assessment or prediction techniques.

Despite nearly fifty years of research and development it is a sad fact of life that the application and practice of fracture mechanics analysis techniques is still generally confined to large sophisticated organisations such as aerospace companies and bridge builders, both involved with high risk projects. The lack of general awareness of the power and applicability of one or other branches of fracture mechanics in general engineering is not helped by a curious "Catch 22" situation with materials suppliers. It is still customary for metal suppliers to charge handsomely for supplying fracture mechanics data with a batch of material, or even to decline to supply any fracture data other than Charpy data. From a design perspective the fracture toughness of a piece of material is almost as important a material property as the tensile strength although nothing like so easy to incorporate in the design. Until this problem is resolved, catastrophic failures and non-catastrophic fractures will continue to occur with depressing frequency.

One of the subsidiary objectives of this course is to try to demonstrate that most fracture events can be predicted through the application of fracture mechanics with an acceptable degree of certainty and at reasonable cost to the manufacturer or user.

In particular however we want to be able to provide quantitative answers to one or more of the following questions which might be asked of a particular design or material:

1. Given that a crack exists in a component or structure what load can be applied without the crack extending in an unstable manner?
2. Knowing the service loads (design stresses) on a component or structure what is the maximum crack size that the component or structure can sustain without risk of failure?

3. For a component with a preexisting crack how long does it take for that crack to grow from its initial size to a critical size from which fracture may occur?
4. What is the anticipated service life of a component or structure which contains preexisting defects of known size arising from manufacturing defects or material inhomogeneities?
5. For a component or structure with a preexisting defect what frequency of inspection is appropriate to ensure that this defect does not grow to a critical size during operation?

Some of these questions are inter-related and can be posed in different ways but in essence the objective of this course is to provide a framework through which you should be able to answer questions similar to those outlined above.

Chapter 3 Origin of *G*

Griffith approached the understanding of fracture via the concept of surface energy yielding important general results for its analysis.

Energy-based analysis of fracture leads to definition of the **strain energy release rate** to characterise the loading on a crack and of the **critical energy release rate** as a material toughness property.

3

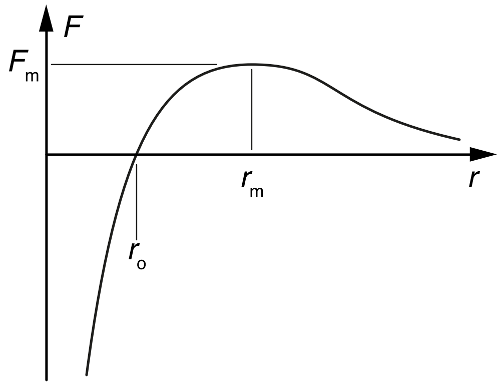
Origin of G

3.1 The theoretical stress approach to fracture

Calculating the stress needed to separate perfect crystal planes.

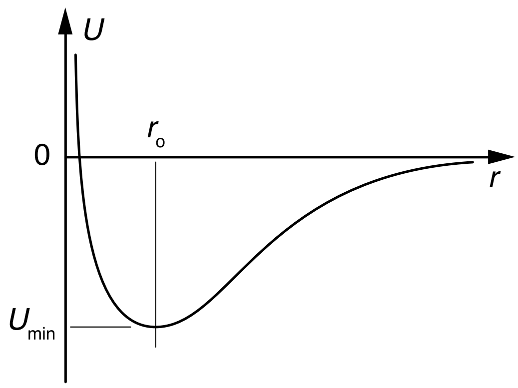
From ME1 Materials we know that two atoms (or ions) can attract or repel each other, but they remain two entities. Attractive forces exist at long range and short range, but repulsive forces are only significant at short range.

Therefore if we could measure the force between two atoms as the distance between them was varied, we would get a net attractive force at large (by atomic standards) distances and a net repulsive force at small distances, with zero force at some equilibrium distance, r_o.



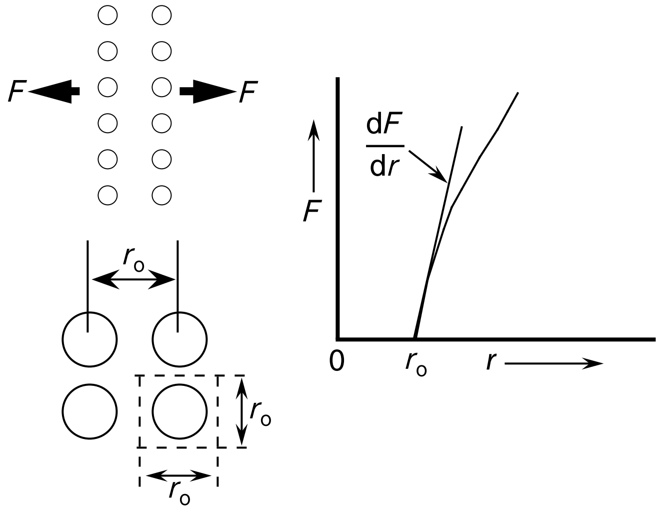
In this context it is also often very useful to consider the potential energy in the bond which is simply given by:

$U = \int F\,dr$ … … … (Eq. 1)



What we want to know is the magnitude of the force and hence the stress required to cause a crystalline solid to fracture across a particular crystallographic plane. To carry out such a calculation we need to consider each atomic bond as a spring element initially at a separation of r_o.

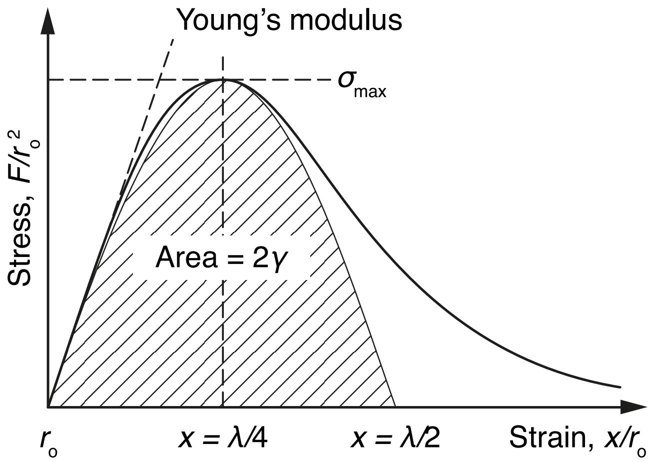
Consider the simplest case of 2 layers of atoms, packed in square, on planes perpendicular to the applied force (Fig 2 below). Consider what happens when we try to pull apart two atoms, one in each layer, so that the interatomic spacing increases from r_o to $r_o + \delta r$:



If the bond is stretched to the max value F_m of F, it will be unstable and will break if the stress continues to increase. Therefore an estimate of the tensile strength could be obtained by calculating F_m. However instead of considering the maximum force F_m we can arrive at a value for the maximum stress directly by making the connection that each atom effectively occupies an area of r_0^2.

To carry out such a calculation we must assume that the force displacement diagram can be approximated by a sine-wave for the part of the F-r curve from r_o to r_m. Also we need to let $(r - r_0) = x$ so that the diagram can be redrawn in terms of stress versus strain where strain is shown directly as x/r_0. The stress is then given by F/r_0^2.

The resultant atomic stress-strain curve appears as shown here.



To separate two atoms from their equilibrium position to infinity the total 'work of fracture' is equivalent to the minimum energy part of the U/r curve at $r = r_o$. This work of fracture is also usually taken as being equivalent to the energy required to create two new surfaces, i.e. the two surfaces of our 'crack' which must occur if we are to separate the two atoms to infinity. The surface energy of the free surfaces created by fracture is denoted by γ and as such the cross hatched area must be equivalent to 2γ. The relationship for the theoretical atomic stress-strain curve is given by

$$\sigma = \sigma_{max}\sin\left(2\pi\frac{x}{\lambda}\right) \dots \dots \dots \text{(Eq. 2)}$$

where λ is the wavelength such that $\sigma = \sigma_{max}$ at $x = \lambda/4$. The total cross-hatched area under the curve represents the 'work to fracture' so we can equate this with the minimum energy U_o. Hence $U_o = 2\gamma$ and then we can see that

$$\frac{\lambda}{2\pi}\sigma_{max}\left[-\cos\left(\frac{2\pi x}{\lambda}\right)\right]_0^{\lambda/2} = 2\gamma$$

Now we also know that for small displacements $\sin x \approx x$ so

$$\sigma = \sigma_{max}2\pi\frac{x}{\lambda} = E\frac{x}{r_0}$$

and substituting for λ we get

$$\frac{\sigma_{max}^2 r_o}{E}[2] = 2\gamma$$

or re-arranging in terms of σ_{max} we have the theoretical fracture stress:

$$\sigma_{max} = \sqrt{\frac{E\gamma}{r_o}} \dots \dots \dots \text{(Eq. 3)}$$

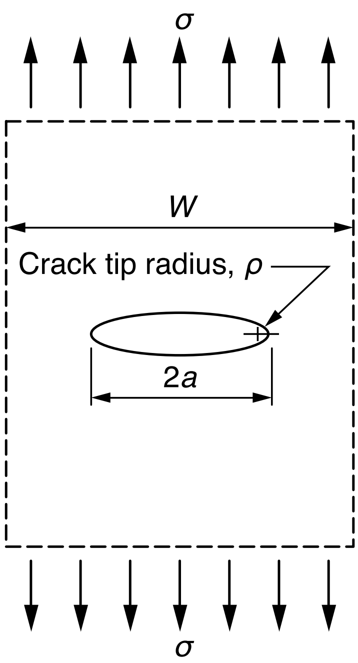
In reality γ is of the order of $r_o/100$ such that σ_{max} has a value of $\approx E/10$ which is at least one to two orders of magnitude higher than we observe in practice. The theoretical approach is somehow fatally flawed — but how ?

When all else fails try a different approach.

3.2 The energy balance approach

Thermodynamic analysis of fracture as a progressive process of separation rather than an all-at-once event.

In essence Griffith proposed that the presence of a sharp crack or flaw in a body might be a sufficient stress concentrator that the theoretical strength could be reached in some small, localised area in that body. To get a first approximation of what the stress would need to be let us consider a semi-elliptic defect with a crack tip radius ρ lying normal to the applied stress in a body:



According to a solution due to Inglis the stress at the ends of the crack is given by

$$\sigma = \sigma\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

As the crack tip radius becomes infinitely sharp we can assume that ρ approaches the lattice spacing r_o, hence

$$\sigma \approx 2\sigma\sqrt{\frac{a}{r_o}}$$

and by assuming that fracture occurs when $\sigma = \sigma_{max}$ we have

$$\sigma_{max} = 2\sigma\sqrt{\frac{a}{r_o}} = \sqrt{\frac{E\gamma}{r_o}}$$

or in terms of the fracture stress, σ_f we get that

$$\sigma_f = \sqrt{\frac{E\gamma}{4a}} \ldots\ldots\ldots \text{(Eq. 4)}$$

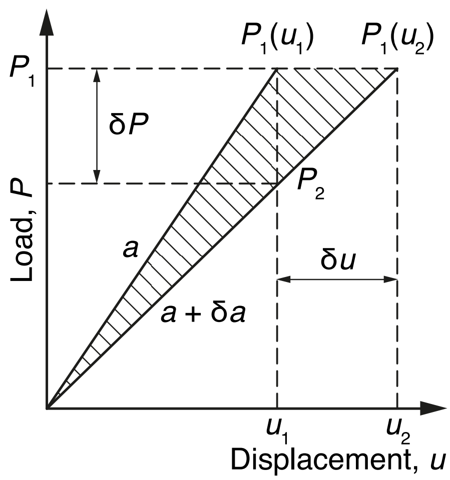
If we put a few numerical values in, as before, we can see immediately that a defect length of even a few microns is sufficient to lower the fracture strength by two orders of magnitude. This approach therefore looks attractive but has serious limitations in that it assumes too much by taking a totally elastic solution (Inglis) to predict a stress which experience tells us is far from elastic.

Griffith overcame these objections by re-casting this approach in thermodynamic terms in order to get away from having to consider any of the crack tip processes.

Consider an infinite plate containing a central, through thickness crack of length 2*a* and subjected to a remotely applied uniform tensile stress of σ. We are in the habit of drawing this configuration as above despite having just stated that the plate is infinite in the *x*-*y* plane. What we really mean is that the boundaries of the plate are sufficiently far removed from the ends of the crack that the fracture process cannot be influenced by the boundaries.

Now consider what happens to the total energy of the system as we extend the crack by an infinitesimal amount. As the crack extends two new crack surfaces are created and these surfaces have an associated surface energy of γ/unit area. Thus the energy required to achieve our increment of crack growth is simply 2γ times the area of the new crack surfaces.

Now imagine it were possible to produce a load-displacement diagram for the condition where the crack length is *a* and then superimpose the diagram for the condition when the crack length is $a + \delta a$. Under fixed grip conditions the diagram would appear as below.



As the crack extends the stiffness of the plate will decrease such that because the grips are fixed (equivalent to fixed displacement, i.e. constant u_1) the load applied by the grips will decrease as the crack extends. In energy terms we can see that for crack length *a* the elastic strain energy is given by $\frac{1}{2}P_1u_1$ and that this changes to $\frac{1}{2}P_2u_1$ as the crack extends to $a + \delta a$.

Hence under fixed grip conditions the extension of the crack from a to $a + \delta a$ results in the release (decrease) of elastic strain energy from the plate equivalent to $\frac{1}{2}(P_1 - P_2)u_1$.

This release of stored elastic strain energy must go somewhere and intuitively it would not seem unreasonable for this energy to be consumed in the work of fracture required to create the two new crack surfaces. Before further discussing precisely where and how this elastic strain energy is consumed we should consider what happens under fixed loading conditions since this represents the other end of the spectrum to the assumption of fixed grips or constant displacement.

Although a bit more complicated, the same principles apply as for fixed grips. As the crack grows the plate effectively becomes a weaker spring and we need an increase in displacement to keep the load constant. The stored strain energy for the crack of length $a + \delta a$ is $\frac{1}{2}P_1 u_2$, which is apparently greater than for the crack of length a which has an associated elastic strain energy of only $\frac{1}{2}P_1 u_1$. However we have to remember that in achieving the increase in crack length the applied load P has moved through a distance $u_2 - u_1$ or put another way we have done work $P_1(u_2 - u_1)$ on the system. Thus the overall change in potential energy of the system is still a decrease expressed as

$$\Delta U_E = P_1(u_2 - u_1) - \frac{1}{2}P_1(u_2 - u_1) = \frac{1}{2}P_1(u_2 - u_1) \ldots \ldots \ldots \text{(Eq. 5)}$$

and this quantity of released potential energy is equivalent to the cross hatched area in the load/displacement diagram above.

Now compare the energy released for fixed load with the fixed grip condition. Since we can simplify matters by defining

$$\delta u = (u_2 - u_1)$$

and

$$\delta P = (P_1 - P_2)$$

it can be seen that:

$$\text{strain energy release (fixed grip)} = -\frac{1}{2}\delta P u \ldots \ldots \ldots \text{(Eq. 6)}$$

and

$$\text{potential energy release (constant load)} = -\frac{1}{2}P\delta u \ldots \ldots \ldots \text{(Eq. 7)}$$

Also we need to consider the relationship between load and displacement in the general case. As for any elastic system the displacement and load are related through a simple linear equation such that for any given crack length we can write that

$$u = CP \ldots \ldots \ldots \text{(Eq. 8)}$$

where C is a constant referred to as the compliance of the system. (Note that C has the inverse units to stiffness since compliance is in effect the inverse of stiffness)

As the increment of crack length $\delta a \to 0$, the value of C will tend to be the same for a crack of length a and one of length $a + \delta a$. We can therefore re-write Eq. (8) as

$\delta u = C\delta P \ldots\ldots\ldots$ (Eq. 9)

and substituting this into Eqs. (6) and (7) we get

$$-\frac{1}{2}\delta\, Pu = -\frac{1}{2}CP\,\delta P$$

and

$$-\frac{1}{2}P\delta u = -\frac{1}{2}CP\,\delta P$$

In short:

Note

There is no difference in the energy released when an infinitesimally small increment of crack growth occurs under conditions of fixed load (potential energy) or conditions of fixed grips (elastic strain energy).

Griffith made the important connection in recognising that the driving force for crack extension is the energy which can be released and that this is used up as the energy required to create the two new surfaces. This thermodynamic description of the fracture process has the huge advantage of removing attention from the small area at the crack tip and the precise micromechanism of fracture.

Without going into rigorous detail, Griffith arrived at the following relations for the change in energy of a body with crack length as follows:

$$U = -\frac{1}{2}\frac{\sigma^2\pi a^2}{E}$$

and

$$\frac{\delta U}{\delta a} = -\frac{\sigma^2\pi a}{E} = G \ldots\ldots\ldots \text{(Eq. 10)}$$

for plane stress and

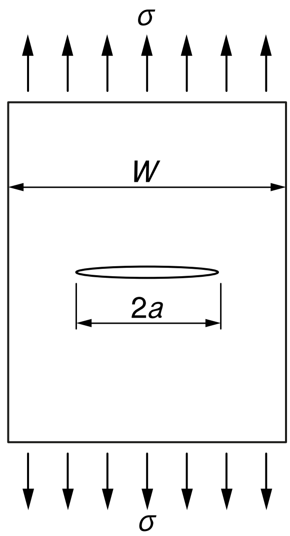
$$U = -\frac{1}{2}\frac{\sigma^2\pi a^2}{E}\left(1-\upsilon^2\right)$$

and

$$\frac{\delta U}{\delta a} = -\frac{\sigma^2\pi a}{E}\left(1-\upsilon^2\right) = G \ldots\ldots\ldots \text{(Eq. 11)}$$

for plane strain. The problem is therefore how we get from these equations relating energy, applied stress and crack length to some practical relationship which predicts the onset of fracture as a function of crack length and applied stress.

We have seen that $\delta U/\delta a$ represents the decrease in potential energy of a system when a crack extends by a small amount δa under constant load conditions.



Consider a body containing a crack of half length a and subjected to an externally applied stress, σ. The total energy of the system can be thought of as being the sum of the potential energy term, U plus the surface energy of the crack, S, thus

$$\text{Total energy} \quad U + S = -\frac{1}{2}\frac{\sigma^2 \pi a^2}{E} + 2\gamma a \ldots\ldots\ldots \text{(Eq. 12)}$$

and the maximum in the total energy will occur when

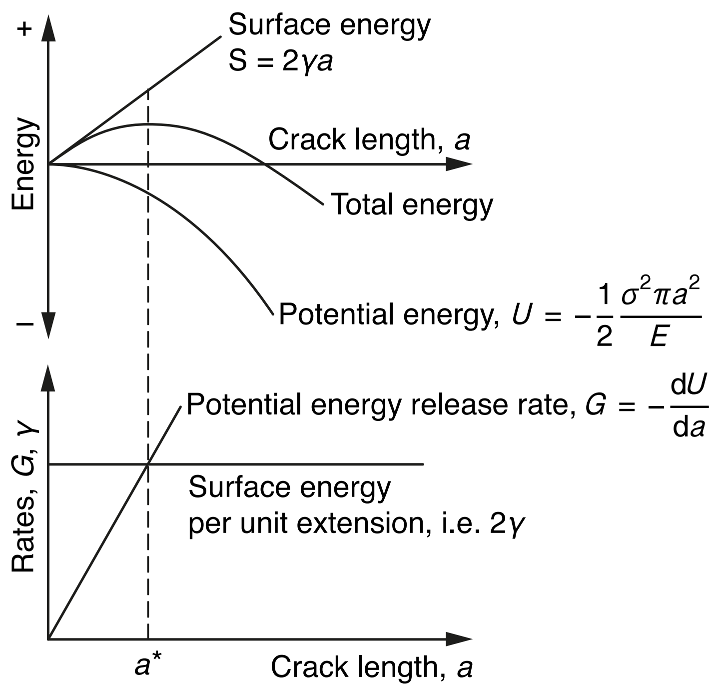
$$\frac{\sigma^2 \pi a}{E} = 2\gamma \ldots\ldots\ldots \text{(Eq. 13)}$$

i.e. crack growth instability will occur when the variation of total energy with crack length reaches a maximum.

Thus the instability condition for crack growth can be expressed as:

$$\frac{\sigma^2 \pi a}{E} \geq 2\gamma \ldots\ldots\ldots \text{(Eq. 14)}$$

The fracture criterion can be visualised by considering how the individual components of the energy and the total system energy vary with crack length. For the situation described above, the energy and rate diagrams would appear as shown below:



Fracture is deemed to occur when the potential energy release rate, G exceeds the surface energy per unit crack extension which must be provided to the system if crack growth is to occur. Where these two lines meet the crack length is the **critical** or **Griffith crack length**, a^*.

Thus by re-arranging Eq. (14) it is possible to show from that for a given crack length a the fracture stress σ is given by :

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}} \ldots\ldots\ldots \text{(Eq. 15)}$$

under plane stress and

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a\left(1-\nu^2\right)}} \ldots\ldots\ldots \text{(Eq. 16)}$$

under plane strain conditions.

In reality these relations are only the basis for further extrapolation since these two equations are really only valid for truly brittle materials such as glass. For quasi-brittle materials such as metals we have to understand that the γ term for the surface energy of the two new crack faces is very small in relation to the plastic work occurring at the crack tip during the actual fracture event. Thus we need to modify these relations before we can apply them to the problem of fracture in materials, which are not classically brittle.

Although the Griffith equations above are not directly applicable to fracture in more ductile materials the inherent inverse-square relationship between crack length and fracture stress is found to be generally applicable and the Griffith approach correctly predicts the effect of flaws or defects in reducing fracture strengths of a material. The development of fracture mechanics applicable to other classes of material has therefore taken the basic thermodynamic argument of Griffith and merely changed the definition of how or where the energy in the system is calculated.

3.3 Calculating or measuring *G*

Energy analysis provides a method for calculating the crack extension force in a specific loaded geometry.

We know from Eqs. (10) and (11) in 3.2 that :

$$U = -\frac{1}{2}\frac{\sigma^2 \pi a^2}{E} \text{ and}$$

$$\frac{\delta U}{\delta a} = -\frac{\sigma^2 \pi a}{E} = G \text{ for plane stress, and}$$

$$U = -\frac{1}{2}\frac{\sigma^2 \pi a^2}{E}\left(1 - \upsilon^2\right)$$

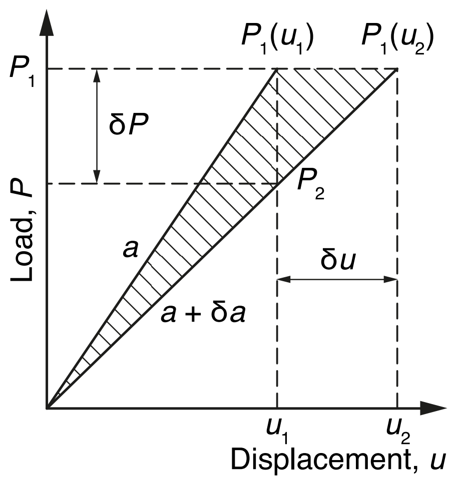
and

$$\frac{\delta U}{\delta a} = -\frac{\sigma^2 \pi a}{E}\left(1 - \nu^2\right) = G$$

for plane strain.

The problem is to determine *G* either analytically or by experiment such that we have an input value in order to predict the fracture stress of a supposed cracked body (or to predict the critical crack length if the applied load is known) through appropriate manipulation of Eqs. (13) and (14).

Again if we look at our elastic loading diagram for crack lengths a and $a + \delta a$ we can hopefully see an experimental method by which we could determine G.



We know that the strain or potential energy release for an increment of crack growth δa is given by $G\delta a$ per unit thickness and if we define B as the thickness of the plate we can say that:

3 Origin of G

$$G\delta aB = \frac{1}{2}P\delta u \ldots\ldots\ldots \text{(Eq. 17)}$$

This quantity (the hatched area in the figure) is not easy to measure as δa tends to zero, so invoking the compliance relationship in Eq. (6) we can rearrange the relationship as follows:

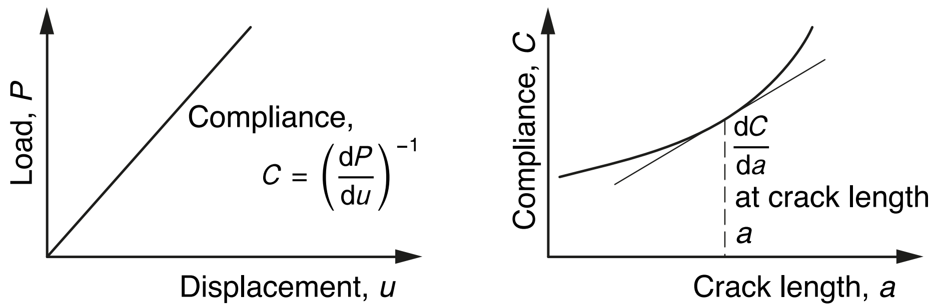
$$GB\delta a = \frac{1}{2}P^2\delta C \ldots\ldots\ldots \text{(Eq. 18)}$$

and as δa tends to zero we have that:

$$G = \frac{P^2}{2B}\frac{\partial C}{\partial a} \ldots\ldots\ldots \text{(Eq. 19)}$$

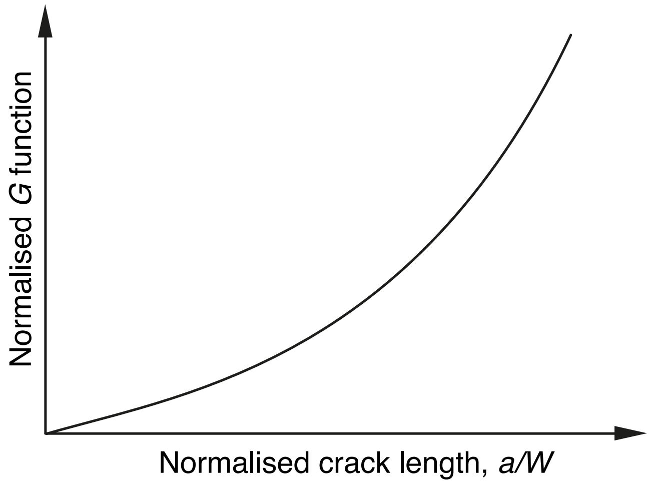
and this important relation can be shown to hold for both fixed grip and constant load situations.

Hence there are many situations, especially for small test pieces, where G can be found as a function of crack length by determining the compliance function for the specimen through experiment. A specimen of known dimensions is initially loaded elastically to measure its compliance at effectively 'zero' crack length. The specimen then has the crack length increased either by machining or through controlled fatigue crack growth and at each increment of crack growth the elastic compliance is determined. A graph of compliance as a function of crack length is then plotted and $\delta C/\delta a$ is found for each increment of crack length either by drawing tangents to the compliance-crack length curve or by numerical techniques.



Caution

Although the tangent method is a tedious procedure, faster numerical methods must be rigorously thought through. Most data curve fitting routines use polynomials which cannot be simply differentiated to give a monotonic function since they are by definition nth order expressions which differentiate to $n - 1$ order expressions.



Having successfully determined the compliance function for a particular geometry an energy release rate function can then be derived as a function of crack length which can be used to calculate G_c (the critical value of strain energy release rate or "fracture toughness") in subsequent tests using specimens of the same geometry.

For large structures it is not easy or practical to measure compliance and in this situation it is appropriate to invoke numerical analysis in order to calculate G as a function of load and crack length. Also for small laboratory scale specimens it is a sensible precaution to check an experimentally determined compliance calibration against an existing or new numerical solutions since it is all too easy to determine a compliance/crack length curve which includes the compliance of the loading train of the testing machine rather than purely that of the specimen.

3.4 *G* for quasi-brittle materials

Griffith developed his theory for classically brittle materials and it was left to later workers in the field to extend his work to other materials.

The first attempts to apply a Griffith fracture stress equation to the problem of unstable fracture in a 'tough' material such as a thin aluminium sheet showed that the relationship between fracture stress and the square root of $E/\pi a$ was maintained but that the constant product of E (cf Eqs. (13) and (14)) was much larger than the classical surface energy term, 2γ.

Orowan and Irwin independently suggested that the disparity was due to the fact that for quasi-brittle materials a great deal of plastic deformation energy was consumed prior to and during the fracture event at and near the crack tip. Thus the thermodynamic balance of Griffith should be modified to include a plastic work term γ_p as follows:

$$\sigma_f = \sqrt{\left(\frac{E\left(2\gamma + \gamma_p\right)}{\pi a}\right)} \quad \ldots\ldots\ldots \text{(Eq. 20)}$$

but since $\gamma_p \gg \gamma$ the surface energy term can be ignored altogether with no great loss of accuracy, hence:

$$\sigma_f = \sqrt{\left(\frac{E\gamma_p}{\pi a}\right)} \ldots \ldots \ldots \text{(Eq. 21)}$$

This simplification has attractions since, by carrying out a few simple tests with cracks of differing lengths, a value for γ_p can be found directly and used as a means of predicting the onset of brittle fracture under other conditions of applied load and crack length. In practice this type of procedure is rarely carried out since the conditions under which reproduceable fracture toughness results can be measured are better understood and implemented for the stress intensity factor approach to fracture.

As will be demonstrated later it can be shown that the strain energy release rate approach to prediction of crack instability through G_c and the complementary idea of the critical value of stress intensity factor are simply related and are thus to a large extent interchangeable. If anything, the concept of a critical value of strain (or potential) energy release rate is a more rigorous approach which gives a better model of the fracture process but is rather less easy to manipulate than the equivalent idea of a critical value of stress intensity at fracture.

Chapter 4 Origin of *K*

The analysis of fracture can also be approached via elastic stress analysis of the region around a sharp-tipped crack.

4.1 Brittle fracture

The characteristics of brittle fracture and examples of materials which demonstrate it.

Materials which can exhibit Brittle Fracture

- Mild steel at low temperature
- High strength Fe, Al and Ti alloys
- Glass
- Perspex
- Ceramics
- Concrete
- Fresh Carrots

Characteristic features of a Brittle Fracture

1. Very little general plasticity — broken pieces can be fitted together with no obvious deformation.
2. Rapid crack propagation ($V_s/3$), eg , ~1000 m/s for steel
3. Low failure load relative to general yield load.
4. Low energy absorption.
5. Usually fractures are flat and perpendicular to maximum principal stress.
6. Fracture always initiates at a flaw or a site of stress concentration.

4.2 Stress concentration at notches

The dependence of stress concentration factor on notch length and tip radius.

Low stress brittle fractures occur in components which are loaded **elastically** but contain flaws from fabrication or service (eg porosity, fatigue, corrosion, etc). The fracture instability must therefore be associated in some way with the **concentration of stress** in the vicinity of the flaw.

The maximum elastic stress at the tip of an elliptical notch of depth a and root radius ρ is given by the Inglis solution which we have already seen:

$$\sigma_{max} = \sigma_{nom}\left(1 + 2\sqrt{\frac{a}{\rho}}\right) \cong 2\sigma_{nom}\sqrt{\frac{a}{\rho}} \;\ldots\;\ldots\;\ldots\; \text{(Eq. 1)}$$

In traditional engineering design such stress concentrations are denoted by the stress concentration factor k_t:

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \text{ ... ... (Eq. 2)}$$

Caution

Do not confuse the stress concentration factor k_t with the stress intensity factor K or K_{Ic}. Typical situations where stress concentrations arise are changes of section or point loads. For such situations the use of appropriate k_t values in design is essential, particularly in fatigue design. (k_t values for all commonly occurring notch geometries are available in Petersen, *Stress concentration factors*).

4

Origin of K

4.3 Stress concentration at cracks

For an infinitely sharp crack the SCF is infinite, and must be redefined.

For a sharp crack the radius of the tip is extremely small (typically 10^{-3} to 10^{-5} mm). Equation (1) predicts that $\sigma_{max} \to \infty$ as $\rho \to 0$. This suggests that for a sharp crack, any applied stress will cause infinitely high stresses at the tip. Also for very sharp cracks, this approach cannot distinguish between long and short cracks whereas experience tells us that failure stress depends on crack length.

Conclusion: the concept of stress concentration factor breaks down as crack tip radius tends to zero.

Consider instead the product of σ_{max} at the crack tip and the crack tip radius, ρ:

$$\frac{\sigma_{max}}{2}\sqrt{\pi\rho} \text{ ... ... (Eq. 3)}$$

(Numerical factors 2 and π are introduced for later convenience). If we substitute for σ_{max} from Eq. (1) and allow $\rho \to 0$, Eq. (3) reduces to:

$$\sigma_{nom}\sqrt{\pi a} \text{ ... ... (Eq. 4)}$$

This remains finite and is formed from the physical quantities σ_{max} and a which define the problem. This is a definition of the stress intensity factor K.

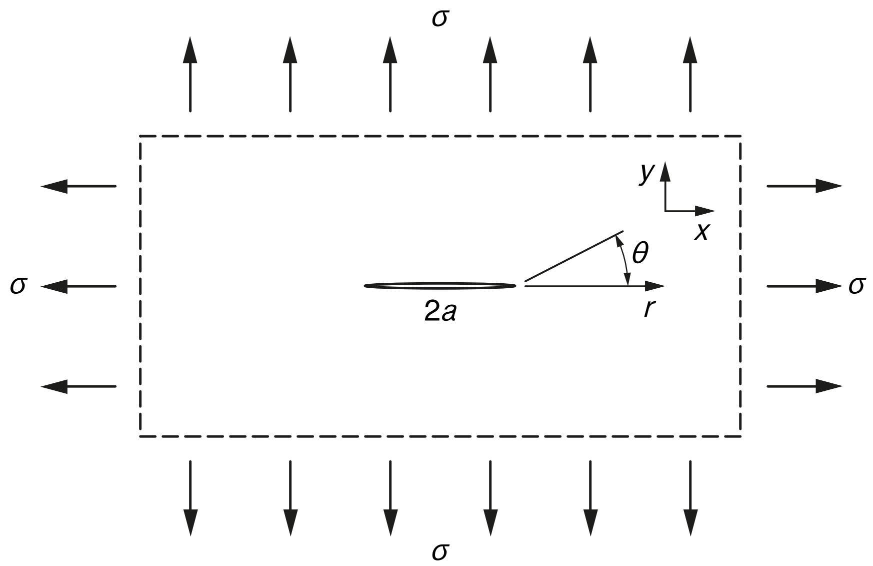
4.4 Stress-strain fields ahead of a crack

Short description.

The preceding loose definition of K results from considering the stress concentration effect of a crack in somewhat general terms. A more rigorous analysis for the stress analysis of a crack is included here for completeness.

The solutions that follow were compiled in 1948 and became known as the Westergaard solutions. The expressions for the stresses are obtained neglecting plasticity in the first instance, i.e. a material is assumed which has no elastic limit.

Consider an infinite plate with a through-thickness centre crack of length 2*a* and subjected to a uniform, biaxial state of stress (Fig. 4.1).



Defining a cylindrical coordinate system with the origin at the tip of the crack, the stresses can be written as a function of the radial distance, *r*, and the angle, *θ*:

$$\sigma_{xx} = \left(\frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\right)\cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \left(\frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\right)\cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] \ldots\ldots\ldots \text{(Eqs. 5)}$$

$$\tau_{xy} = \left(\frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

Equations (5) indicate that all the stresses tend to infinity as the radial distance tends to zero crack tip), i.e. A stress singularity exists at the crack tip.

The equations also show why cracks usually continue to propagate in the plane of the starter crack since σ_y is maximum for $\theta = 0$.

4.5 Plasticity and triaxiality

In most real materials, a loaded material is blunted by plastic deformation.

The above solution gives an idea of how powerful a stress concentrator a crack can be, but it assumes a material which can respond elastically up to infinitely high stress levels. It is quite obvious that no material possesses such properties and, in reality, some amount of plastic response must be present near the crack tip. Moreover it is quite usual to find cases of brittle fracture (macro) with MVC as a micromechanism and that implies some plasticity at the crack tip (the very essence of MVC). However, this plasticity also must be somewhat limited, since cracking occurs before general yield.

The next step is try to describe the extent and geometry of a plastic zone which could exist in a stress field described by the Westergaard solutions.

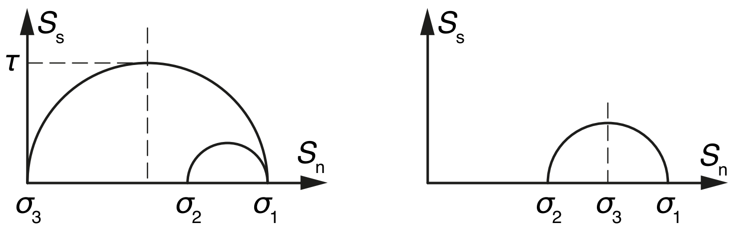
As usual in plasticity analyses, description of the field in terms of principal stresses facilitates visualisation. The principal stresses for the same Westergaard field are:

$$\left.\begin{aligned}\sigma_1 &= \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right)\\ \sigma_2 &= \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)\end{aligned}\right\} \ldots\ldots\ldots \text{(Eq. 6)}$$

Tresca or Von Mises may be used as yield criteria for metals and they are both indicative of the presence of shear (shear strain energy in the case of Mises and maximum shear stress in Tresca's case), therefore some assumption must be made as to the value of the third principal stress component, σ_3. Two possibilities are of interest:

1. Plane stress: $\sigma_3 = 0$;
2. Plane strain: $\epsilon_3 = 0$ hence $\sigma_3 = \nu(\sigma_1 + \sigma_2)$.

In plane stress, as the radial distance r decreases, the magnitude of σ_1 and σ_2 increases whereas σ_3 remains zero. The shear stress component can therefore increase quite dramatically. For the plane strain situation, σ_3 is related to the sum $(\sigma_1 + \sigma_1)$ and therefore is also singular. The shear stress component becomes limited, as illustrated by the Mohr's circles.



Note: the figure for plane strain does not follow the usual $\sigma_1 > \sigma_2 > \sigma_3$ convention.

The plane strain state is closer to a hydrostatic stress state and as a consequence has a high degree of triaxiality associated with it. In practice, plane stress is a reasonable description of the stress state in thin sheets, whereas plane strain describes the stress state of thick components. For the latter situation stresses rise sharply near the crack tip resulting in a small plastic zone surrounded by elastic material, since the elastic Poisson's ratio is about 0.3 whereas the plastic ratio tends to 0.5, and the plastic zone tends to contract (or dimple) much more intensely than the surrounding elastic material. This mismatch results in what is usually called constraint and much higher stresses can be achieved before the material deforms plastically (**contained yield**).

Thick sections are then usually associated with the terms plane strain, high constraint, high triaxiality, contained yield and higher severity.

Important

The stress analysis of a crack explains how brittle fracture (macro) can occur (regardless of micromechanism) when plasticity is contained by an increased level of triaxiality. LARGE THICKNESS implies PLANE STRAIN TRIAXIALITY implies LESS PLASTICITY implies BRITTLE FRACTURE.

4.6 The stress intensity factor and the shape factor

The definition of a single scaling factor to represent the loading geometry and configuration.

Inspection of the equations which describe the stress field ahead of the crack tip indicates that all of the expressions are of the form

$$\underbrace{\frac{1}{\sqrt{2\pi r}}}_{(a)} \times \underbrace{f(\theta)}_{(b)} \times \underbrace{\sigma\sqrt{\pi a}}_{(c)}$$

- Term (a) represents the singularity since this term $\rightarrow \infty$ as the radial distance, $r \rightarrow 0$.
- Term (b) describes the variation with respect to the angle θ and is limited (consider the trigonometric functions).
- Term (c) is a simple function of remote stress σ and crack length a. This simple function dictates the intensity or the magnitude of the stress field and is called the **stress intensity factor**, K.

Important

The stress intensity factor, K, is a single parameter which completely specifies the amplitude of the stress field in the vicinity of the crack tip.

For the initial case of the infinite plate subjected to uniform stresses, the stress intensity factor is written

$K = \sigma\sqrt{\pi a} \ldots \ldots \ldots$ (Eq. 9)

which is justifiable since, for an infinite plate, the only known dimension is the crack length.

In general, the stress intensity factor depends on the geometry of the cracked body (including the crack length) and it is usual to express it as

$K = Y\sigma\sqrt{a} \ldots \ldots \ldots$ (Eq. 10)

where Y is called the shape factor and is a function of body geometry and crack length.

Caution

The $\sqrt{\pi}$ term is included in the Y factor, but some authorities and textbooks refer to the infinite plate case (Eq 9) and leave the fundamental $\sigma\sqrt{\pi a}$ term outside the Y. You will see both definitions!

4.7 Modifications for real geometries

Short description.

The above expression for K is applicable to an ideal situation of an infinite plate containing a centre crack of length $2a$. In practice the presence of finite boundaries and the way in which the crack is loaded affects the value of the stress intensity factor.

- **Loading geometry**

A crack can be loaded in three ways.

Mode I

has the crack opening under the influence of a stress at right angles to the crack plane;

Mode II

involves in-plane sliding normal to the crack front under the influence of a shear stress parallel to the crack plane;

Mode III

involves in-plane sliding parallel to the crack front under the influence of a shear stress parallel to the crack plane.

Stress intensities can be defined for each type of displacement and are designated K_I, K_{II} and K_{III}. However, the great majority of failures occur under mode I loading and hence most fracture toughness data relate to this mode of loading. For mode I loading, the critical stress intensity factor is designated K_{Ic}.

- **Component geometry**

The geometry of the cracked body imposes an effect on the near crack tip stress field and hence modifies the value of the stress intensity factor. For real geometries K is defined above as:

$K = Y\sigma\sqrt{a}$

Y is the geometrical factor which is dependent on crack length and specimen dimensions.

A commonly encountered Y factor is for a straight-fronted edge crack. If the crack length a is small in proportion to the component depth, then $Y = 1.99$, i.e. the open edge allows K to increase by 12%.

1. For a centre cracked plate of finite width:

   $K_I = Y\sigma\sqrt{a}$

   $Y = \left(\sec\frac{\pi a}{W}\right)^{1/2}.$

   So… given the width W, the applied stress σ and the crack length a, the applied stress intensity factor K_I can be calculated.

2. For an edge notch in tension:

   $K_{Ic} = Y\sigma\sqrt{a}$

   $Y = 1.99$ for **small** cracks, otherwise:

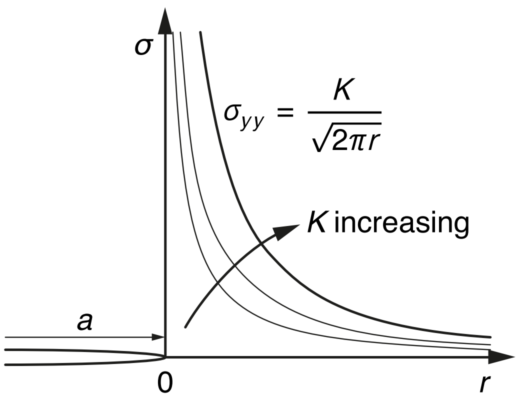
   $Y = 1.99 - 0.41\left(\frac{a}{W}\right) + 18.7\left(\frac{a}{W}\right)^2 - 38.48\left(\frac{a}{W}\right)^3 + 53.85\left(\frac{a}{W}\right)^4.$

Y factors for most other crack geometries are available.

4.8 Critical stress intensity factor K_{Ic}: a fracture criterion

Any response of a crack to applied load — including unstable fracture — is determined by the magnitude of K.

The only parameter distinguishing one crack situation from another is K. Therefore if fracture is controlled by conditions in the vicinity of the crack tip, failure must be associated with the attainment of a critical stress intensity: K_{Ic}.



At fracture

$K_c = Y\sigma_c\sqrt{a}$ … … … (Eq. 11)

In practice we may attain K_{Ic} by increasing either σ or a. For a defect of fixed length a, σ_f is the critical value of the applied stress for fracture. Alternatively, for a constant applied stress σ, a_c is the critical defect size for fracture.

K_{Ic} is a macroscopic criterion for failure. It makes no assumptions about the precise mechanism of fracture.

Note

K is a stress field parameter independent of the material, whereas K_{Ic} is a material property, the **fracture toughness**. (Compare stress σ which can have any value, and σ_y which is a specific material property).

The units of *K* are (stress)√(distance). This may be written most clearly as MPa√m but usually appears as MN $m^{-3/2}$ or occasionally as N $mm^{-3/2}$.

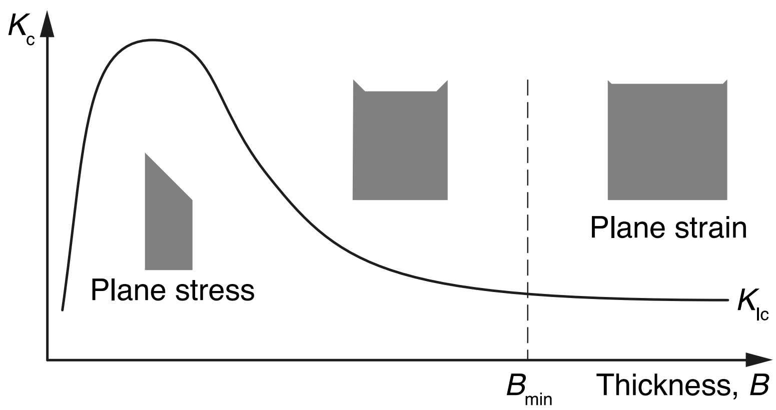
Important

The stress used in evaluating the *K* parameter, and in all fracture mechanics calculations, is the nominal stress acting on the body in the vicinity of the defect; it must not take into consideration any loss of section caused by the presence of the defect.

4.9 Component thickness

Due to crack-tip plasticity, the value of K_c is influenced by component thickness.

The restricting hypothesis is that elasticity adequately describes the stress/strain field. Since the thickness plays a major role in the validity of the elastic description (large thickness, contained yield etc.), the thing to do is to look at how the critical *K* varies with thickness for a certain material. If K_c is measured as a function of specimen thickness, a typical curve drawn through the experimental results would appear as:



Above a certain thickness B_{min}, the fracture toughness has a minimum value which is independent of thickness. This minimum value of K_c is known as the plane strain fracture toughness and is denoted by K_{Ic}. Particular attention is paid to K_{Ic} because this is the minimum toughness that can be achieved under the most severe conditions of loading.

The minimum specimen thickness required to ensure the plane strain conditions necessary for K_{Ic} is given by:

$$B_{min} = 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 \dots\dots\dots \text{(Eq. 12)}$$

As the specimen thickness decreases from B_{min}, the fracture toughness K_{Ic} increases until a maximum value is attained. This maximum reflects the attainment of fully plane stress conditions. Thereafter the toughness again decreases for very small thicknesses.

The changes in K_{Ic} with thickness are accompanied by corresponding changes in fracture geometry. In the plane strain regime the fracture surface is oriented at 90° to the direction of loading (ie "square" fracture). As the thickness decreases, 45° "shear lips" appear on either side of a flat central regime. At and below the thickness corresponding to the maximum K_{Ic} position, the shear lips occupy the full thickness and one has a 45° "shear" or plane stress fracture.

Note

For any thickness of material, the fracture may be completely brittle in the engineering sense, irrespective of the fracture surface geometry or whether plane stress or plane strain loading conditions prevail.

4.10 Inter-relationship of K_c and G_c

Short description.

The strain energy approach to brittle fracture gives:

$$\sigma_f = \sqrt{\frac{E2\gamma}{\pi a}} = \sqrt{\frac{EG_C}{\pi a}}$$

for plane stress conditions and

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a\left(1-\nu^2\right)}} = \sqrt{\frac{EG_{1C}}{\pi a\left(1-\nu^2\right)}}$$

(G_{Ic} = plane strain critical strain energy release rate) for plane strain conditions. Substituting for K from Eq. (9) gives the relationship at brittle fracture:

$K_c^2 = EG_c$ under plane stress loading conditions ……… (Eq. 13)

(or in the general case $K^2 = EG$ for plane stress loading) and

$K_{1c}^2 = EG_{1C}$ under plane strain loading conditions ……… (Eq. 14)

where

$$G = \frac{\pi\sigma^2 a}{E}$$

is the elastic energy release rate and is named after Griffith. G has dimensions of energy per unit area (cracked area).

It follows that a measured value of K_{1c} can be converted to a value of G_{Ic} and vice-versa, the main underlying assumption being that linear elasticity describes the stress/strain field adequately. Both G_{Ic} and K_{Ic} are called the fracture toughness of a material.

4.11 Stress ahead of the crack tip

The component of local stress acting across the crack line is singular, but the other components show different forms.

So far we have considered the elastic stress normal to the crack plane, i.e. $\sigma_{yy} = \frac{K}{\sqrt{2\pi r}}$. Note σ_{yy} is in the same direction as σ_{nom} and arises from the stress concentrating effect of the crack.

What about stresses in x and z directions?

- **The *x*-direction**

  The region of stress intensification is confined to a small region ($\approx a/10$) ahead of crack tip. Outside this region, the material experiences the nominal elastic stress, σ_{nom}.

  Consider an element of material located within the highly stressed region which is exposed to $\sigma_{yy} = \frac{K}{\sqrt{2\pi r}}$.

  If treated as an isolated, uniaxial tensile specimen, it would extend elastically to give a strain commensurate with the local stress, ie $\epsilon_{yy} = \frac{\sigma_{yy}}{E}$ and contract laterally by $-\upsilon\varepsilon$, the Poisson contraction.

  But because the surrounding material in the x direction stops the contraction, ie a σ_x stress develops due to **constraint**.

  At a blunted crack tip we have a small area of free surface normal to the x direction, hence at this position $\sigma_{xx} = 0$. However, as we enter the crack tip zone σ_{xx} tends to σ_{yy}.

  Ahead of the crack tip,

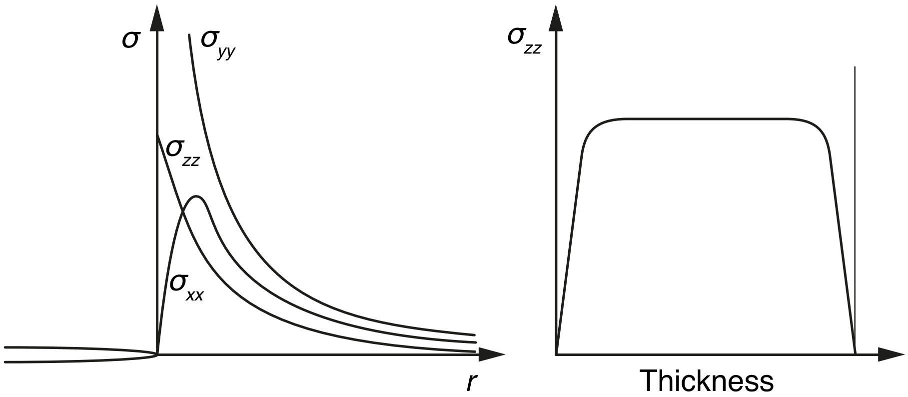
  $$\sigma_{xx} = \sigma_{yy} = \frac{K}{\sqrt{2\pi r}}$$

- **The *z*-direction**

  For thin sheet there is **zero stress** in the thickness direction, i.e. $\sigma_{zz} = 0$ but there will be a thickness strain which manifests itself as a local sucking in or 'notch root contraction'. This condition is known as **plane stress**.

For very thick material there will be no strain in the thickness direction due to the constraining influence of the surrounding lightly stressed material, i.e. $\varepsilon_{zz} = 0$.

This condition is known as **plane strain**. Although there is no thickness strain in plane strain, there must be a positive thickness stress to resist the Poisson contraction.



Generalised Hookes Law:

$$\epsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

Hence for plane strain, $\varepsilon_{zz} = 0$

$$\sigma_{zz} = \nu\left(\sigma_{xx} + \sigma_{yy}\right).$$

4.12 Crack tip plasticity

Yield criteria can be applied to investigate the extent of plastic flow near a loaded crack tip.

LEFM assumes that the material is **elastically** loaded, i.e. there is negligible **plastic** deformation of the material. For ceramic materials, the fracture stress is a very small proportion of the yield stress and the assumption of elastic behaviour is justified. For other materials the situation is not so clear.

The K-field associated with the crack predicts that the local stress σ_{yy} tends to infinity as the distance ahead of the crack tip tends to zero. Consequently for metallic and polymeric materials, the stresses immediately ahead of the crack tip must exceed the yield strength of the material and a local region of plasticity must develop. The extent of the plastic zone depends on the yield strength of the material and the state of stress at the crack tip.

- **Plane stress plastic zone size**

  In plane stress $\sigma_{zz} = 0$. Hence yield can occur when $\sigma = \sigma_y$ (Tresca).

  Therefore a first estimate of plastic zone size can be obtained by substituting the yield stress for σ_{yy}

$$\sigma_y = \sigma_{yy} = \frac{K}{\sqrt{2\pi r_p}}$$

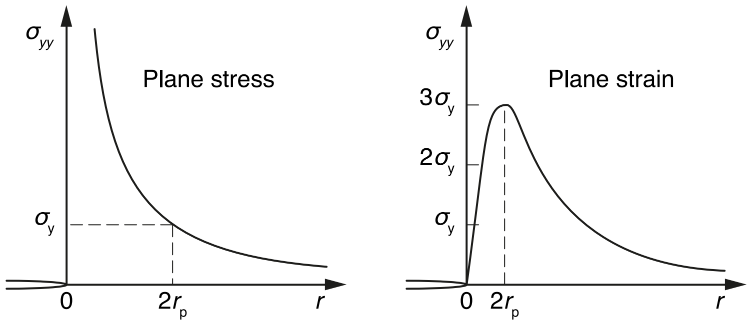
$$\text{i.e. } r_p = \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2 \quad \ldots\ldots\ldots \text{(Eq. 15)}$$

However, redistribution of stress must occur to account for the lost load carrying capacity under the truncated part of the stress distribution. In fact the plastic zone must extend to $2r_p$ to accommodate the lost load. Therefore r_p is the radius of the crack tip plastic zone.

- **Plane strain plastic zone size**

  Strain in thickness direction $\varepsilon_{zz} = 0$, but σ_{zz} is positive.

  Therefore ahead of the crack tip, σ_1, σ_2 and σ_3 are all positive and yielding cannot occur until $\sigma_1 > \sigma_y$ due to triaxial constraint.



Consider the situation ahead of the crack tip where $\sigma_1 = \sigma_{yy}$ and $\sigma_2 = \sigma_{xx}$ but $\sigma_{yy} = \sigma_{xx}$:

$$\sigma_3 = \sigma_{zz} = \nu\left(\sigma_{xx} + \sigma_{yy}\right).$$

Thus for e.g. $\nu = 0.3$, $\sigma_3 = 0.6\sigma_1$.

Hence using the Von Mises criterion:

$$\sqrt{2}\sigma_y = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

whence $\sigma_1 = 2.5\sigma_y$, i.e. the local stress ahead of the crack tip must exceed $2.5\sigma_y$ in order to achieve yielding under conditions of triaxial constraint.

More detailed analysis predicts that for a non-work hardening material, the maximum stress in the plane strain plastic zone is approximately $3\sigma_y$. For a work hardening material, maximum local stress in the plastic zone may attain 4 to 5 times σ_y.

The consequence of the increased stress for yielding under conditions of triaxial constraint is that the plastic zone size in **plane strain** is smaller than in plane stress. The simple analysis used for plane stress is **not** applicable to the more complex 3D stresses in plane strain but it can be shown that the plane strain plastic zone size is approximately 1/3 of that in plane stress and is given by:

$$r_p = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_y}\right)^2 \ldots\ldots\ldots \text{(Eq. 16)}$$

Note

The above expressions for r_p have used K and K_I rather than K_c and K_{Ic}. This is because there will be a plastic zone associated with the crack for any applied stress intensity K. When the K value reaches the critical value, K_c, the plastic zone will have achieved its maximum size. Thus for plane strain,

$$r_p = \frac{1}{6\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2 \ldots\ldots\ldots \text{(Eq. 17)}$$

- **Crack length plasticity correction**

In the presence of crack tip plasticity, the elastic stress field (i.e. that part represented by K) can be more accurately characterised by considering an effective crack of length $(a + r_p)$. Hence, for situations of limited plasticity, we should strictly incorporate a **plasticity correction** into the LEFM equations by defining an effective crack length

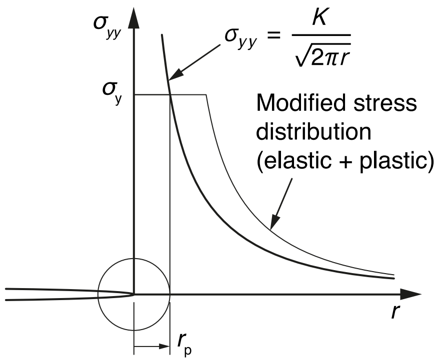
$$a_{eff} = a + r_p \ldots\ldots\ldots \text{(Eq. 19)}$$

whence

$$K = Y\sigma\sqrt{\pi\left(a + r_p\right)} \ldots\ldots\ldots \text{(Eq. 20)}$$

What is the extent of the load redistribution due to the limited plasticity, and can we estimate it more precisely?

Consider the following diagram.



As we move in towards the crack tip σ_{yy} increases until we reach a point where the yield strength σ_y is exceeded. The cross hatched area below the σ_{yy} here represents one part of the contribution to the load carrying capacity but there is also the area above this which patently cannot sustain stresses much above the yield stress value and therefore leads to a loss of load carrying capacity. Hence this loss of load carrying capacity must be compensated for by having larger stresses directly ahead of the crack tip.

Therefore our problem is how to estimate the extent of the increase in stresses beyond the crack tip due to load redistribution.

We know that the area under the curve can be determined by integrating the expression for σ_{yy} between limits of r_p and zero. So, to a first approximation we ignore the load carrying contribution of the cross hatched area completely to get

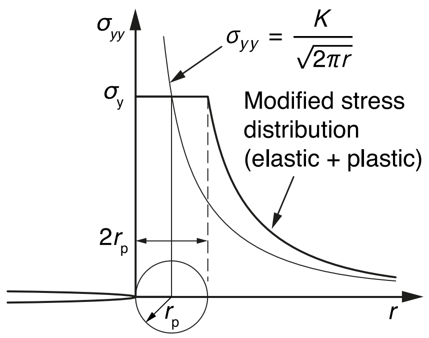
$$\text{Area} = \int_0^{r_p} \sigma_{yy} = \int_0^{r_p} \frac{K}{\sqrt{2\pi r}} = \frac{2K}{\sqrt{2\pi}}\left[r^{1/2}\right]_0^{r_p} = \frac{2Kr_p^{1/2}}{\sqrt{2\pi}}$$

and we know that the plane stress plastic zone size is given by

$$r_p = \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2$$

and substituting this expression for r_p gives:

$\text{Area} = \frac{2K}{\sqrt{2\pi}}\frac{K}{\sigma_y}\frac{1}{\sqrt{2\pi}} = \frac{2K^2}{2\pi\sigma_y} = \frac{K^2}{2\pi\sigma_y^2}2\sigma_y = 2r_p\sigma_y$ i.e. the extent of the load redistribution under plane stress conditions is such that, to a first approximation, the plastic zone influence extends to twice the apparent extent.



4.12.1 Plastic zone size at fracture

Examples for several engineering materials.

- **High strength steel**

  $\sigma_y = 1200\ \text{MN m}^{-2}$

  $K_{Ic} = 60\ \text{MN m}^{-3/2}$

  Plane strain $r_p = 0.13\ \text{mm}$.

  Size of plastic zone occupies about 5 grain diameters, i.e. very localised plasticity and the linear elastic assumption is reasonable.

- **Perspex**

  $\sigma_y = 30\ \text{MN m}^{-2}$

$K_{Ic} = 1\ \mathrm{MN\,m^{-3/2}}$

Plane strain $r_p = 60\ \mu\mathrm{m}$.

Still a very small value in comparison with dimensions of sheet etc. Therefore LEFM reasonable.

- **Alumina**

  $\sigma_y = 5000\ \mathrm{MN\,m^{-2}}$

  $K_{Ic} = 1\ \mathrm{MN\,m^{-3/2}}$

  Plane strain $r_p = 2\ \mathrm{nm}$.
  Extent of any plasticity would be comparable with interatomic dimensions, i.e. effectively non-existent.

- **Structural steel**

  $\sigma_y = 400\ \mathrm{MN\,m^{-2}}$

  Plane strain $K_{Ic} = 150\ \mathrm{MN\,m^{-3/2}}$

  For thick sections $r_p = 7.5\ \mathrm{mm}$.

  Plane stress $K_c = 250\ \mathrm{MN\,m^{-3/2}}$

  For thin sections $r_p = 60\ \mathrm{mm}$

  (Must question whether LEFM assumptions are valid for large plastic zones in lower strength metallic materials).

4.12.2 Consequences of crack tip plasticity on toughness

Because toughness is related to plastic surface work, its value is partly determined by yield stress/strain properties.

The development of a plastic zone is by far the major source of energy consumption in a fracture process, i.e. crack tip plasticity is the source of toughness. This is why ceramics and high strength metals have poor toughness.

Remember that:

- $K_{Ic}^2 = EG_{Ic}$ and $G_{Ic} = 2\gamma_s$;
- The true surface energy $\gamma_s = 2\ \mathrm{J\,m^{-2}}$; and
- For metallic materials $G_{Ic} = 2(\gamma_s + \gamma_p) \approx 10^4\ \mathrm{J\,m^{-2}}$.

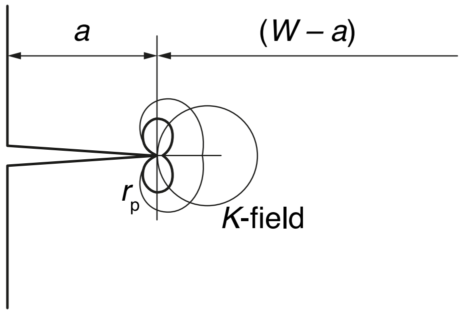
- **Plane strain toughness**

  1. Small plastic zone in relation to sample thickness causes fracture surface to be oriented at right angles to applied stress, i.e. a **square** fracture.

2. Small plastic zone as compared with plane stress contributes to lower toughness in plane strain.

3. High stress in plastic zone ,$\approx 3\sigma_y$ encourages low energy cleavage mode of fracture which requires attainment of local stresses in excess of σ_y (n.b. only applicable to materials which are capable of cleavage).

4. High stress triaxiality encourages void nucleation and growth for fibrous mode of fracture.



- **Plane stress toughness**

1. Large plastic zone in relation to thickness causes local shearing at 45° through sample thickness, ie a slant fracture.

2. Large plastic zone causes large energy consumption and high K_{Ic}.

3. Stress in plastic zone limited to uniaxial yield stress σ_y. Therefore more difficult to satisfy micro fracture criteria.

4. In steel, cleavage is impossible under plane stress conditions because micromechanism of fracture requires local stresses considerably in excess of σ_y.

- **Minimising constraint**

To ensure fully plane stress conditions there must be no triaxial constraint ahead of the crack tip. This condition is achieved when the plane stress plastic zone is greater than the sample thickness, i.e.

$$2r_p = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2 > B.$$

Local slip can then take place on ±45° planes through the full thickness of the sample, ie $\sigma_{zz} = 0$, therefore no constraint.

May deliberately try and achieve this condition by increasing K_{Ic} or decreasing B in order to avoid the risk of low energy cleavage fracture in mild steel.

Note

This strategy does not necessarily eliminate the risk of brittle fracture because the crack could still propagate by a low energy fibrous mechanism.

4.12.3 Constraints on LEFM validity

A summary of the limits within which LEFM assumptions are justifiable.

1. Characterisation of elastic crack tip stresses by K is valid only for the region immediately ahead of the crack tip, i.e. for $a/10$ ahead of tip.
2. Embedded within the elastic field is the plastic zone:

   $r_p = \frac{1}{2\pi}\left(\frac{K_c}{\sigma_y}\right)^2$ for plane stress.
3. For events in the plastic zone to be controlled by the surrounding K field, i.e. for crack tip processes to be controlled by K, the plastic zone size must by less than 20% of the K field, ie . $r_p < a/50$.
4. Similar contraints apply to the next section below the crack ($W - a$) ie $r_p < (W - a)/50$.
5. Conclude that for LEFM (plane stress and plane strain):

   $a, (W - a) > 50r_p$.
6. For plane strain there is an additional requirement that the size of the plastic zone must be less than 1/50 of the thickness B, ie $B > 50r_p$.

   $r_p = \frac{1}{6\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$ for plane strain so that:

   $a, (W - a), B > 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$.

4.12.4 Practical implications of crack tip plasticity

The applicability of LEFM to some specific structural materials.

- **High strength and/or brittle materials**

  For high strength and brittle materials the plastic zone size is small, and the macroscopic fracture behaviour is dominated by the elastic stresses in the material.

  1. High strength aluminium alloy
     $K_{Ic} = 25\ \mathrm{MN\,m^{-3/2}}$

     $\sigma_y = 500\ \mathrm{MN\,m^{-2}}$

     Plane strain $r_p = 0.13\ \mathrm{mm}$

     For LEFM plane strain $B_{min} = 6\ \mathrm{mm}$.
  2. High strength steel

     $K_{Ic} = 60\ \mathrm{MN\,m^{-3/2}}$

$\sigma_y = 1500 \text{ MN m}^{-2}$

$r_p = 0.10 \text{ mm}$

For LEFM plane strain $B_{min} = 5 \text{ mm}$.

3. PMMA (Perspex)

   $K_{Ic} = 1.5 \text{ MN m}^{-3/2}$

   $\sigma_y = 50 \text{ MN m}^{-2}$

   $r_p = 0.05 \text{ mm}$

   For LEFM plane strain $B_{min} = 2.5 \text{ mm}$.

Conclusion: for these materials LEFM fracture mechanics works OK.

- **Medium strength structural steel**

  $\sigma_y = 450 \text{ MN m}^{-2}$

  $K_{Ic} = 80 \text{ MN m}^{-3/2}$

  Plane strain $r_p = 1.7 \text{ mm}$

  For LEFM plane strain $B_{min} = 85 \text{ mm}$.
  In the light of the large B_{min} value necessary for LEFM plane strain validity it is necessary to consider two different service situations:

- **Applications requiring thick sections**

  Example: Nuclear pressure vessel where the wall thickness may be 200 mm.

  LEFM is then appropriate to the service situation and K_{Ic} values should be used for the prediction of critical defect sizes and specification of acceptable defects.

  The problem then is how to measure K_{Ic} to characterise the fracture toughness of the steel used. For the laboratory test we still need to satisfy the conditions for LEFM plane strain validity: a, $(W - a)$ and $B > 85$ mm.

  This may be possible but would be highly impracticable and very costly. If we use a small specimen, we no longer have plane strain conditions, have extensive plasticity and the toughness measurement in terms of K_{Ic} is no longer meaningful.
  Conclusion: We need an alternative method of measuring toughness.

- **Applications requiring thin sections**

  Application is such that material is used in sections much thinner than B_{min}, e.g. ship's deck or pipeline where the plate thickness may be say 15 mm. For this situation the use of K_{Ic} to predict failure would be excessively conservative. For example, whilst the K_{Ic} value might be 80 MN m$^{-3/2}$, the K_{Ic} value appropriate to a plate thickness of 15 mm might be 240 MN m$^{-3/2}$. This equates to an almost 10-fold increase in critical defect size.

In principle we could determine the K_{Ic} value appropriate to the thickness of the plate. However, during such laboratory testing we would still have to satisfy the conditions for LEFM plane stress validity namely:

a, $(W–a) > 50r_p$.

For K_{Ic} = 240 MN m$^{-3/2}$, r_p = 45 mm, i.e. we would need a test panel at least 2.25 m wide. Again testing is impracticable and we need an alternative method of assessing toughness.

4

Origin of K

4.12.5 Material fracture toughness data

Data from Ashby and Jones, Engineering Materials 1.

All values are at room temperature except *.

Material	**G_c (kJ m^{-2})**	**K_{Ic} (MN m$^{-3/2}$)**
Pure ductile metals e.g. Cu, Al, Ni, Ag	100–1000	100–350
Rotor Steels	220–240	204–214
Pressure Vessel Steels e.g. HY 130	150	170
High Strength Steels	15–118	50–154
Mild Steel	100	140
Ti Alloys (Ti 6Al 4V)	26–114	55–115
GFRP	10–100	20–60
Fibreglass	40–100	42–60
Al Alloys (high–low strength)	8–30	23–45
CFRP	5–30	32–45
Woods, crack normal to grain	8–20	11–13
Boron fibre epoxy composites	17	46
Medium C steel	13	51
Polypropylene	8	3
Polyethylene (low–high density)	6–7	1-2
ABS Polystyrene	5	4
Nylon	2–4	3
Cast Iron	0.2–3	6–20
Polystyrene	2	2
Woods, crack parallel to grain	0.5–2	0.5–1
Polycarbonate	0.4–1	1.0–2.6
Cobalt/WC cermets	0.3–0.5	14–16

Material	**G_c (kJ m^{-2})**	**K_{Ic} (MN m$^{-3/2}$)**
PMMA	0.3–0.4	0.9–1.4
Epoxy	0.1–0.3	0.3–0.5
Granite	0.1	3
Polyester	0.1	0.5
Silicon nitride, Si_3N_4	0.1	4–5
Beryllium	0.08	4
Silicon carbide, SiC	0.05	3
Magnesia, MgO	0.04	3
Cement/concrete not reinforced	0.03	0.2
Marble, Limestone	0.02	0.9
Alumina, Al_2O_3	0.02	0.6
Soda glass	0.01	0.7–0.8
Electrical Porcelain	0.01	1
Ice	0.003	0.2*

4.13 Practical application of LEFM

The basis of the fracture toughness approach to design against brittle fracture is that the material property K_c is measured in the laboratory for a convenient specimen geometry and is then applied to a different (practical) geometry to predict the failure stress or critical crack size.

4.13.1 Application to design

Examples of the application of LEFM to avoid fracture where σ_{app} and K_{Ic} are known.

- **Example: Aircraft fuselage**

  Material: Al alloy (eg 2024).

Property	**Value**	**Units**
Yield strength, σ_y	200	MN m^{-2}
$\sigma_{nom} = \sigma_{app} = 0.5\sigma_y$	100	MN m^{-2}
K_c (thin sheet)	120	MN m$^{-3/2}$ (measured in lab)
Y parameter	1.99	
Critical defect size, $a_{crit} = \left(K_c/Y\sigma_{nom}\right)$	0.36	m

Conclusion: crack will propagate unstably if length exceeds 0.36 m (e.g. Comet windows, Trident wings).

- **Example: Aircraft undercarriage**

  Material: HS steel.

Property	Value	Units
Yield strength, σ_y	1200	MN m^{-2}
$\sigma_{nom} = 0.6\sigma_y$	720	MN m^{-2}
K_{Ic} (thick plate)	60	MN m$^{-3/2}$ (measured in lab)
Y parameter	1.86	
Critical defect size, $a_{crit} = (K_c/Y\sigma_{nom})$	1.99	mm

  Conclusion: defect larger then 2 mm will cause catastrophic brittle failure.

- **4.20.3 Example: Plastic ruler**

  Material: Perspex (4 mm thick).

  Design requirement: ruler must bend through radius of 0.25 m

  From beam theory,

  $$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}.$$

Property	Value	Units
Yield strength, σ_y	50	MN m^{-2}
Young's modulus, E	3.5	GN m^{-2}
Hence σ_{max}	28	MN m^{-2}
K_{1c}	1.5	MN m$^{-3/2}$
Y parameter (straight-fronted crack)	1.99	
Critical defect size, $a_{crit} = (K_c/Y\sigma_{max})$	0.7	mm

  Conclusion: consistent with experience. New rulers have $a_{crit} \ll 1$ mm and hence perform OK. Old rulers have environmental stress cracks 1 mm deep, cause brittle fracture when flexed.

4.13.2 Application to failure analysis

Examples of the application of LEFM to failure analysis, where K_{Ic} and the crack size at failure a_{crit} are known.

- **Example: Rudder stock on merchant ship**

  Material: Normalised medium carbon steel.

  Fatigue cracks $a = 16$ mm deep are observed.

Property	Value	Units
Yield strength, σ_y	450	MN m^{-2}
Y parameter for observed crack shape	0.85	
K_{Ic}	40	MN $m^{-3/2}$ (measured in lab)
$\sigma_f = \left(K_c / Y\sqrt{\pi a}\right)$	210	MN m^{-2}

Conclusion: failure stress was 46% of yield stress, which is a normal service stress, i.e. failure was not caused by abnormal loading.

- **Example: Cast iron pipe**

Failed on cold night: temperature = –8 °C, ΔT=–20°C compared to mean temperature. Contained pre-existing crack, 200 mm long. Was failure caused by low temperature or some other source of loading?

$\epsilon = \alpha\Delta T$ and $\sigma = E\epsilon\ldots$

Property	Value	Units
$\sigma = E\alpha\Delta T$	30	MN m^{-2} (cf tensile strength 270 MN m^{-2})
K_{Ic}	23	MN $m^{-3/2}$ (measured)
Y parameter for part-circumferential crack	1.77	
Fracture stress $\sigma_f = \left(K_{Ic} / Y\sqrt{a}\right)$	29	MN m^{-2}

Conclusion: Failure due to low ambient temperature.

4.13.3 Application of LEFM to quality assurance

Cases where LEFM is used to calculate an acceptable defect size and to plan for its detection.

All materials and structures contain defects, e.g.:

- Surface flaws in glass (~1 μm)
- Cracks in welds (~10 mm)
- Fatigue cracks (~1–1000 mm).

Numerous techniques are available for detecting defects with different sensitivities and applications:

1. Visual
2. Dye penetrant
3. Magnetic particle
4. Ultrasonic
5. Radiography (X and γ rays)

The problems:

- What size of defect can one accept?
- What size must one detect by NDT?

Fracture mechanics offers the only rational basis for specification of tolerable defect size and inspection procedures.

- **Example: Aircraft wing**

Material: Al.

Property	Value	Units
K_{Ic} (thin sheet)	120	MN $m^{-3/2}$
σ_{nom}	100	MN m^{-2}
a_{crit}	0.46	m

Conclusion: No need to detect very small defects — cracks can be detected by visual inspection long before they reach critical size.

(Trident wings = 150 mm long cracks).

- **Aircraft undercarriage**

Material: HSS.

Property	Value	Units
K_{1c}	60	MN $m^{-3/2}$
σ_{nom}	720	MN m^{-2}
$a_{crit} = 0.32\left(\frac{K_{1c}}{\sigma_{nom}}\right)^2$	1.8	mm

Conclusion: Inspect after every heavy landing and NDT every week. Must detect 1 mm crack reliably.

- **Pressure vessel**

Material: Steel.

Property	Value	Units
K_{1c} (thick section)	120	MN $m^{-3/2}$
σ_{nom}	200	MN m^{-2}
$a_{crit} = 0.32\left(\frac{K_{1c}}{\sigma_{nom}}\right)^2$	110	mm

Conclusion: Should be able to detect all potentially damaging manufacturing defects by visual, X-ray and UT.

Note

Must also allow for crack growth by fatigue and/or stress corrosion. These effects can also be calculated by LEFM, see later notes.

4.14 The leak-before-break concept

A very important concept in the safe design of pressure vessels.

In the pressure vessel example above, $a_{crit} = 110$ mm.

If wall thickness = 250 mm then $a_{crit} < t$, i.e. if the crack remains undetected and grows by fatigue, we will eventually get catastrophic brittle fracture — or, put another way, the vessel will explode.

Alternatively, by appropriate choice of steel toughness, stress and thickness, we can arrange that $a_{crit} = 300$ mm. For example by increasing the fracture toughness of the steel to >200 MN $m^{-3/2}$ the critical crack length can be increased to over 300 mm.

Now if an undetected crack undergoes slow growth, it must penetrate the wall thickness before it can achieve a size sufficient for unstable fracture. But penetration of the wall will cause leakage of fluid which leads to a loss of pressure which in turn reduces the stress and lowers the driving force for failure. i.e. vessel leaks before it breaks.

4.15 Proof loading

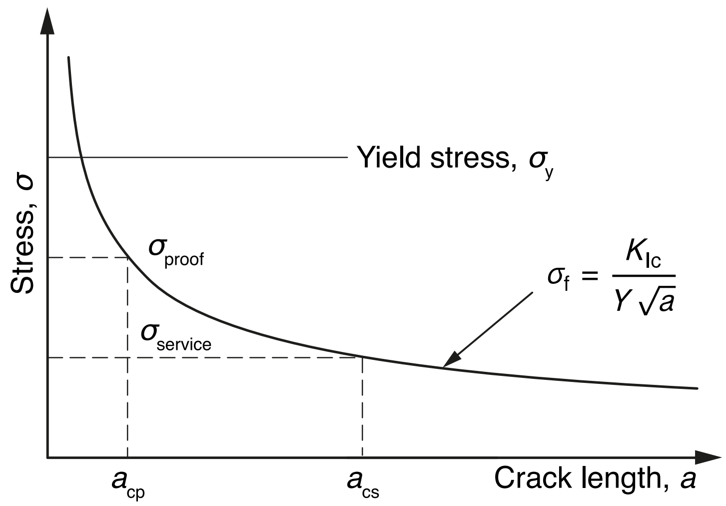
The traditional method of testing engineering components and structures.

Put simply, all that is done is to expose the component or structure to stress > service stress. If it survives it is then considered safe for service at the lower stress.

Can get more information by analysing this process in terms of LEFM. For a service stress $\sigma_{service}$ can calculate a critical defect size = a_{cs}.

If proof load to σ_{proof} and if the structure survives, then you **know** that no defects can be present greater than the critical defect size at σ_{proof} i.e. a_{cp}.

Now you **know** the margin of safety in terms of defect size and if necessary can calculate the time for the defect to grow from the initial size to the critical defect size a_{cs}. This time then provides the basis for determining the subsequent inspection intervals.



4

Origin of K

4.16 Application to materials selection

Some LEFM procedures and standard protocols used to select materials for critical applications.

Defect tolerance may be an important criterion in materials selection. For example in airframe structures, whilst the primary requirement is high specific strength, one must recognise that fatigue cracks will eventually develop and the material must be able to accommodate a crack of reasonable size without failing by an unstable brittle fracture mechanism.

The failure condition is given by

$$K_{Ic} = Y\sigma_{app}\sqrt{a}.$$

Now for a given application, the maximum applied stress, σ_{app} is likely to be some fraction of the yield strength, i.e. $\sigma_{app} = f\sigma_y$ and hence we have a modified failure condition, namely that

$$K_{Ic} = Yf\sigma_y\sqrt{a}$$

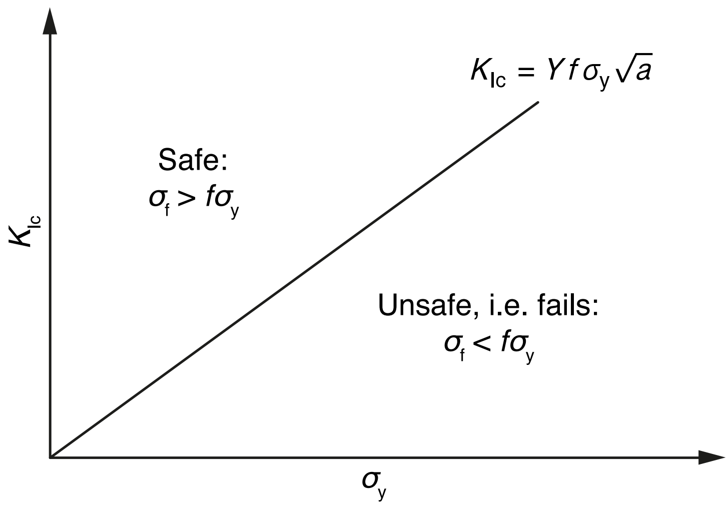
If candidate materials have already been selected, we can calculate and compare a_{crit} values from known values of K_{Ic} and σ_y as above.

Alternatively, if materials are to be selected for equivalent defect tolerance, this can be done by using a **Ratio Analysis diagram**.

4.16.1 Ratio analysis diagram

A graphical method used for selection of steels.

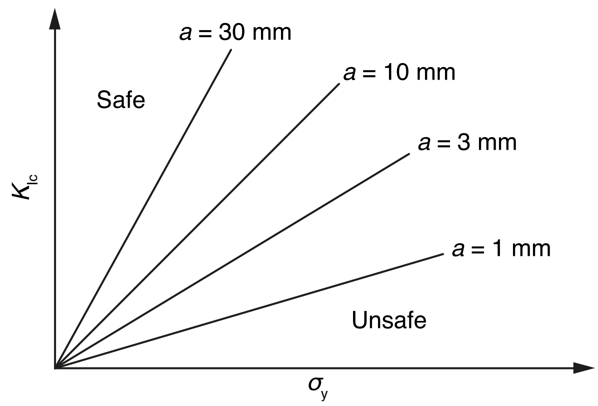
For a given crack size, a, and geometry, Y, the brittle fracture condition can be plotted on a K_{Ic} vs σ_y diagram:



The brittle fracture line divides the diagram into two regions:

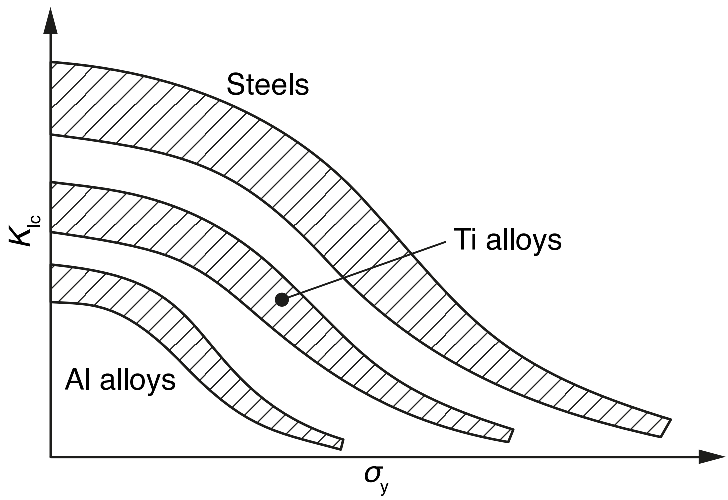
- Above the line σ_f is greater than the applied stress $f\sigma_y$ and hence there is no risk of brittle fracture;
- Below the line σ_f is less than the applied stress and hence brittle fracture is inevitable.

For a range of possible defect sizes, a family of failure lines can be drawn:

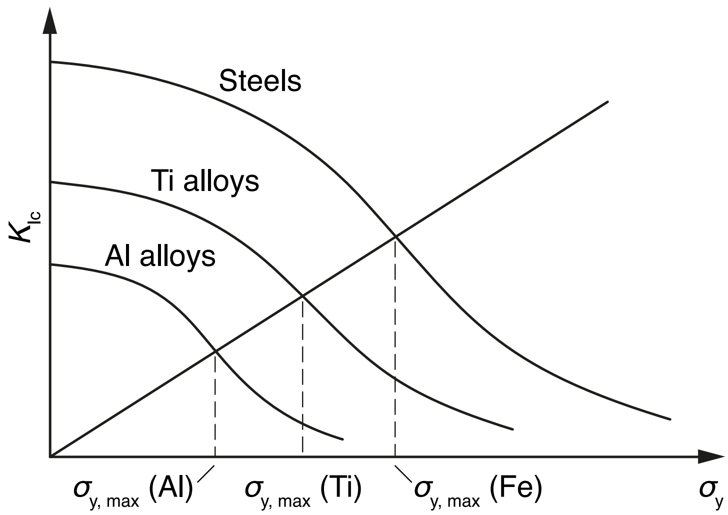


For a particular crack length any combination of fracture toughness and applied stress which puts us to the left of a line implies a 'safe' condition whereas any combination which is mapped to the right of a line implies a potentially 'unsafe' condition. Hence for a particular crack length, the critical failure stress can be easily determined or, for a particular applied stress, the maximum tolerable defect size can be determined.

Actual yield strength versus fracture toughness data for a range of materials can be plotted on K_{Ic} vs σ_y diagrams.

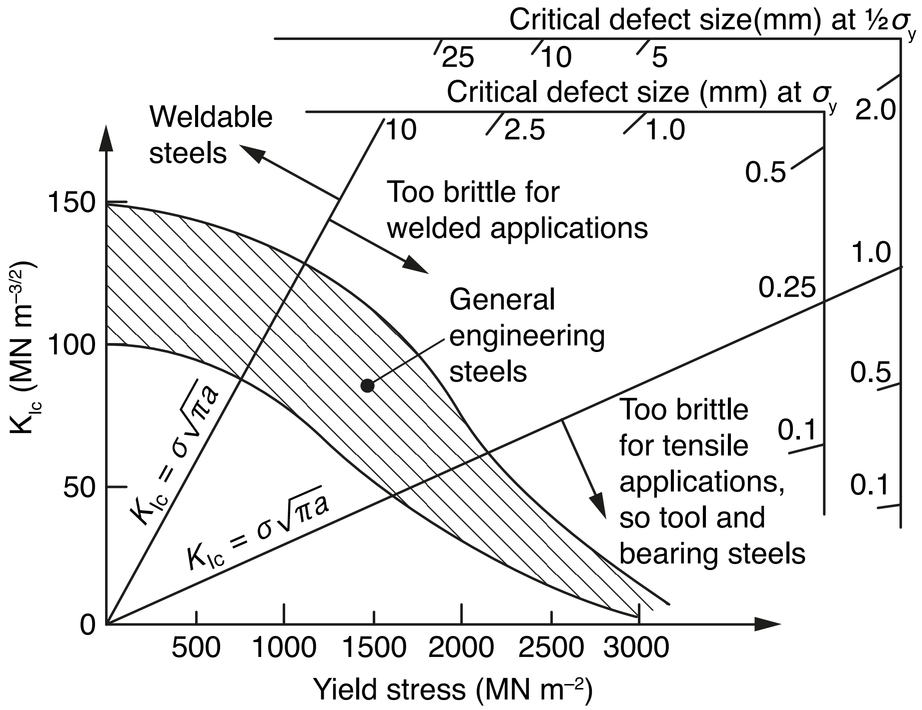


Usually K_{Ic} decreases as strength increases, and K_{Ic} decreases as E decreases (recall $K_{Ic}^2 = EG_{Ic}$). Hence the tendency for different classes of materials to show similar trends in fracture toughness data for the various alloys within that class of material. If we now plot the failure condition on the material property diagram, we can identify the highest strength condition for each class of material which will satisfy the same defect tolerance requirement.



Having identified the highest usable strengths, can then calculate required section sizes and compare weights, costs, etc..

Similarly a Ratio Analysis Diagram can be a useful tool in explaining why it is that materials from within one class of materials are used over a range of strengths or fabricated in different ways for different applications. For steels it can be seen that because the process of welding introduces defects there is a limit to the applied stress and thus by implication to the tensile strength of steels used in structures fabricated by welding.



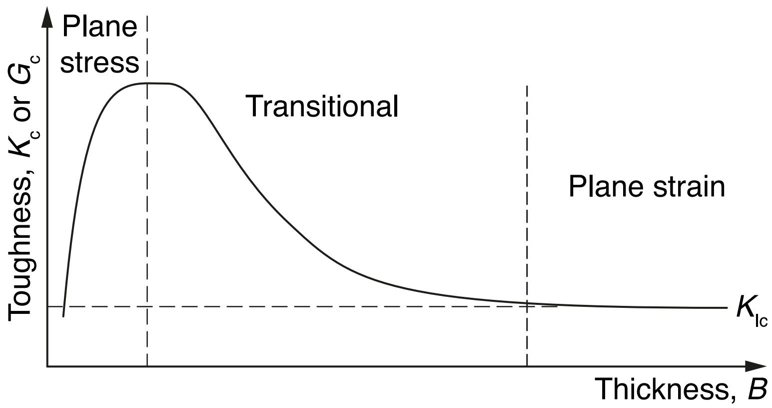
Similarly although very high tensile strength can be achieved in steels by quenching and tempering highly alloyed steels the probability of brittle fracture limits their use in general engineering applications.

Chapter 5 Fracture toughness testing

5.1 Specimen thickness

Because fracture toughness depends strongly on thickness, specimen dimensions must be chosen with care.

The fracture toughness K_{Ic} or for that matter G_c can be a function of test temperature, specimen thickness and the degree of crack tip constraint. This is somewhat unfortunate since in reality most real life crack situations exist under conditions where the stress state is either plane stress or transitional between plane stress and plane strain. Therefore any methodology for fracture toughness testing should in theory be able to cope with both fully plane strain conditions (K_{Ic}) and transitional/plane stress behaviour (K_{Ic}).



Despite this desire no single test procedure has been developed to allow measurement and interpretation of resulting critical values of K or G for all possible stress states. Thus, it is customary to consider plane strain fracture toughness testing as the standard methodology and to consider all other stress conditions as exceptions to this standard or requiring a different approach to interpretation of results.

5.2 Plane strain fracture toughness testing (K_{1c})

For most purposes plane stress properties are regarded as the most conservative benchmark.

The ASTM has devoted much effort to the formulation of testing protocols and development of standard testing methods for the determination of plane strain fracture toughness values in metals and other materials. The current applicable standard for determination of K_{1c} in metallic materials is ASTM E-399-12e3 and this is widely adopted around the world. The equivalent British Standard 7448 of 1993 is a four part document of which all Parts have now been published at various dates and this includes a comprehensive testing methodology for K, G, δ_c (COD) and J.

5

Fracture toughness testing

All standardised fracture toughness protocols include the same elements, namely:

- A list of acceptable specimen geometries;
- Standard specimen preparation techniques;
- Standardised testing procedures; and
- A standardised method of determining *K* or *G* from the resulting test data.

The fact that a particular specimen geometry does not appear in the list of acceptable geometries in e.g. ASTM E-399-12e3 does not preclude its use as a valid means of determining fracture toughness for a particular specimen. The specimen geometries which do appear in the standards are those which have been found by extensive round-robin testing to give reproducable results in different laboratories within about 15%.

- **Specimen Geometry**

  Acceptable specimen geometries are those for which the compliance calibration curve as a function of normalised crack length is well documented and accepted. Given that most practical or potential brittle fracture problems involve fairly large pieces of metal the ASTM and BS both include the Compact Tension (CT) and Single Edged Notched Bend specimen (SENB) which will cater for most available material configurations.

  Since the test procedure requires that a sharp starter crack in the form of fatigue crack is present in the specimen the standards also dictate the geometry of the starter notch from which the fatigue crack has to be initiated and grown to a required length. Specifying the geometry of the starter notch ensures that the fatigue crack initiates and remains in the mid-plane of the specimen and that the crack front is symmetric.

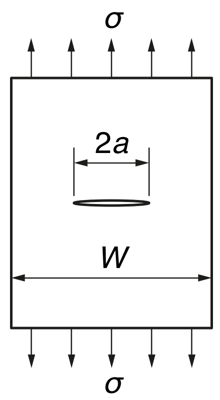
- **Specimen Size**

  Plane strain fracture toughness testing is almost unique in requiring that the experimenter has a good idea of the result before carrying out the test! Since one of the main validity restriction for testing of this type is that fully plane strain conditions are developed at the crack tip the specimen must have certain minimum dimensions.

  Typical geometries which are or have been used for plane strain fracture toughness determination are shown below together with their stress intensity factor solutions and/or *G* solutions.

5.2.1 Centre cracked plate specimen

Geometry and equations for determining K.



$K_I = Y\sigma\sqrt{a}$ where

$$Y = 1.77 + 0.454\left(\frac{a}{W}\right) - 1.99\left(\frac{a}{W}\right)^2 + 21.62\left(\frac{a}{W}\right)^3$$

or

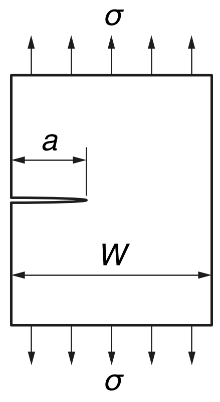
$$Y = \sqrt{\sec\frac{(\pi a)}{W}}$$

or

$$Y = \frac{1}{\sqrt{1-\left(\frac{2a}{W}\right)^2}}.$$

5.2.2 Single edge notched plate geometry

Geometry and equations for determining K.



$K_I = Y\sigma\sqrt{a}$ where

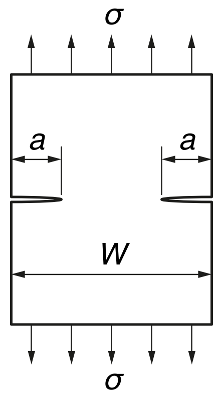
$Y = 1.99$ for short cracks, or

$$Y = 1.99 - 0.41\left(\frac{a}{W}\right) + 18.70\left(\frac{a}{W}\right)^2 - 38.48\left(\frac{a}{W}\right)^3 + 53.85\left(\frac{a}{W}\right)^4$$

up to $a/W = 0.6$.

5.2.3 Double edge notched plate specimen

Geometry and equations for determining K.



$K_{\mathrm{I}} = Y\sigma\sqrt{a}$

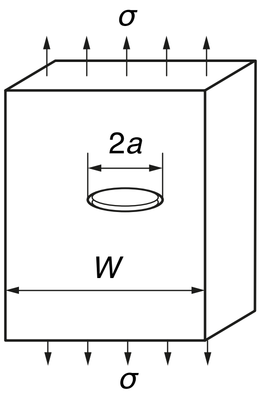
where

Y=1.99 for small cracks or

$$Y = \frac{1.99 - 0.994\left(\frac{a}{W}\right) - 0.363\left(\frac{a}{W}\right)^2 + 0.835\left(\frac{a}{W}\right)^3 - 0.34\left(\frac{a}{W}\right)^4}{\sqrt{1 - \frac{a}{W}}}$$

5.2.4 Embedded penny shaped crack geometry

Geometry and equations for determining K.



$K_{\mathrm{I}} = Y\sigma\sqrt{a}$

where

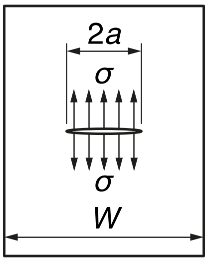
$$Y = \frac{2}{\sqrt{\pi}}.$$

5.2.5 Internally pressurised crack geometry

Geometry and equations for determining K.

5

Fracture toughness testing

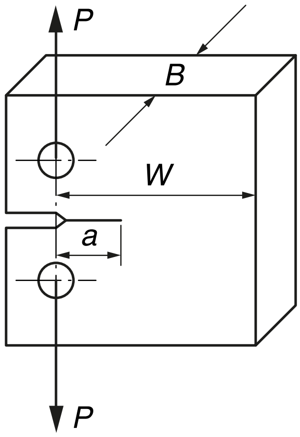


$K_I = Y\sigma\sqrt{a}$ where $Y = P\sqrt{\pi}$

where P is the internal pressure acting on the crack surfaces.

5.2.6 Compact tension (CT) geometry

Geometry and equations for determining K.

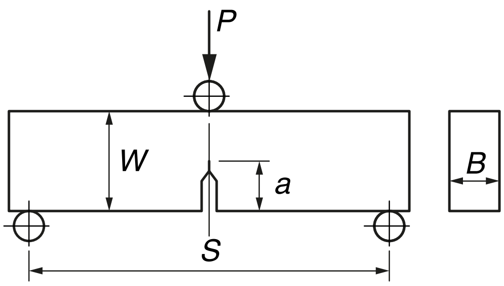


$$K_I = \frac{P}{BW^{1/2}} f\left(\frac{a}{W}\right)$$

$$f\left(\frac{a}{W}\right) = 29.6\left(\frac{a}{W}\right)^{1/2} - 185.5\left(\frac{a}{W}\right)^{3/2}$$
$$+ 655.7\left(\frac{a}{W}\right)^{5/2} - 1017\left(\frac{a}{W}\right)^{7/2} + 638.9\left(\frac{a}{W}\right)^{9/2}$$

5.2.7 Single edge notched bend geometry

Geometry and equations for determining K.



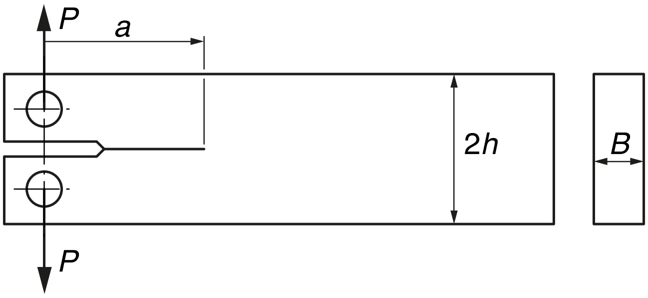
For $S = 4W$, $W = 2B$, $a = B$:

$$K_\mathrm{I} = \frac{PS}{BW^{3/2}} f\left(\frac{a}{W}\right)$$

$$f\left(\frac{a}{W}\right)$$
$$= -37.6\left(\frac{a}{W}\right)^{7/2} + 38.7\left(\frac{a}{W}\right)^{9/2} \; 2.9\left(\frac{a}{W}\right)^{1/2} - 4.6\left(\frac{a}{W}\right)^{3/2} + 21.8\left(\frac{a}{W}\right)^{5/2}$$

5.2.8 Double cantilever beam geometry

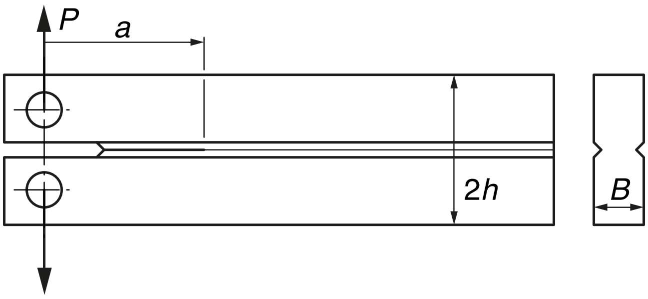
Geometry and equations for determining K for a parallel DCB specimen.



$$K_\mathrm{I} = 2\sqrt{3}\frac{Pa}{Bh^{3/2}} \text{ for plane stress}$$

$$K_\mathrm{I} = \frac{2\sqrt{3}}{\left(1-\nu^2\right)}\frac{Pa}{Bh^{3/2}} \text{ for plane strain.}$$

A recognised difficulty with all beam type fracture toughness specimen geometries is that the crack path has a tendency to deviate from the desired centre plane of the specimen. The most commonly utilised method to avoid crack deviation is to machine side grooves in the specimen.



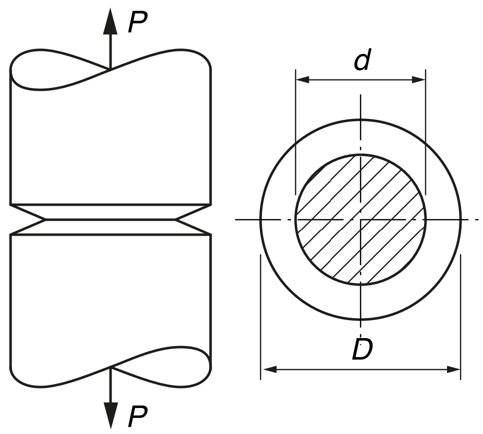
The use of side grooves not only constrains the crack path to follow the mid-plane of the specimen but also provides more constraint at the crack free surface and thereby promotes plane strain conditions, in principle allowing an overall thinner section to be utilised than would have been the case without side grooves.

Attention

It should be remembered that the provision of side grooves will affect the bending stiffness of the arms and that the stress intensity factor expression should be modified to reflect the loss of section.

5.2.9 Circumferentially notched bar geometry

Geometry and equations for determining K for a CNB specimen.



$$K_I = \frac{0.932P\sqrt{D}}{\sqrt{\pi d^2}}$$

which is valid within the range $1.2 \leq \frac{D}{d} \leq 2.1$.

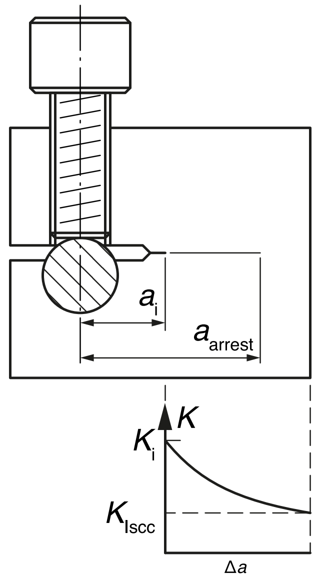
5.2.10 Wedge opening load geometry

The geometry of a WOL specimen, used for crack arrest testing.

There also exist several other geometries of test specimen which have special properties, e.g.:

- They can be used without the benefit of a testing machine, or
- They give a particularly easy method of obtaining a known variation of *K* with crack length.

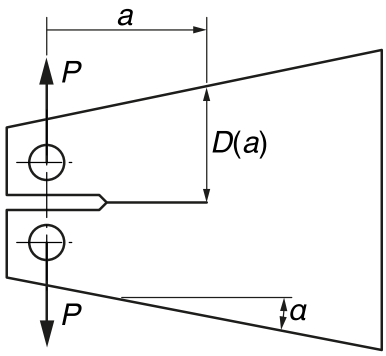
The first of these 'oddities' is the so-called Wedge Opening Load specimen or WOL. This has the advantage of not requiring a testing machine since it is essentially self loading. This specimen type is often used for determination of K_{Iscc} threshold values since the specimen can be loaded to a known value of *K*, immersed in the aggresive environment and left. If crack growth occurs due to stress corrosion then as the crack advances the applied *K* drops since the loading is deflection limited. Thus by determining the crack length when crack arrest occurs a K_{Iscc} value for that material/environment combination may be determined from a single specimen test.



5.2.11 Tapered DCB geometry

Geometry of a TDCB 'constant K' specimen, used for stress corrosion and fatigue crack propagation testing.

It is often useful to have a specimen geometry where G is constant at a fixed load i.e. where K is constant with crack length at fixed load. This could be achieved by having a beam type specimen such as a DCB but with increasing thickness B as a function of crack length. This method of producing a constant K/G specimen is too expensive and hence the alternative strategy is to modify the bending stiffness to achieve constant K/G through addition of material to the depth of the arms in the specimen. Hence the use the contoured cantilever beam which is often also referred to as a 'trouser-leg specimen', or tapered DCB specimen.



This specimen geometry with an angle α of about 12° gives a reasonably constant K over the central section of the specimen but is not truly constant. This type of specimen geometry is most frequently employed when fatigue crack growth rate or environmentally assisted crack growth rate data has to be determined.

5

Fracture toughness testing

5.3 Typical K_{Ic} test procedure

The following list of steps are those which would be most frequently encountered in setting up and carrying out a fracture toughness test. The list is not exhaustive since there may well be special stances as for example by virtue of available material shape or form that precludes the use of an obvious specimen geometry in favour of a non-standard geometry.

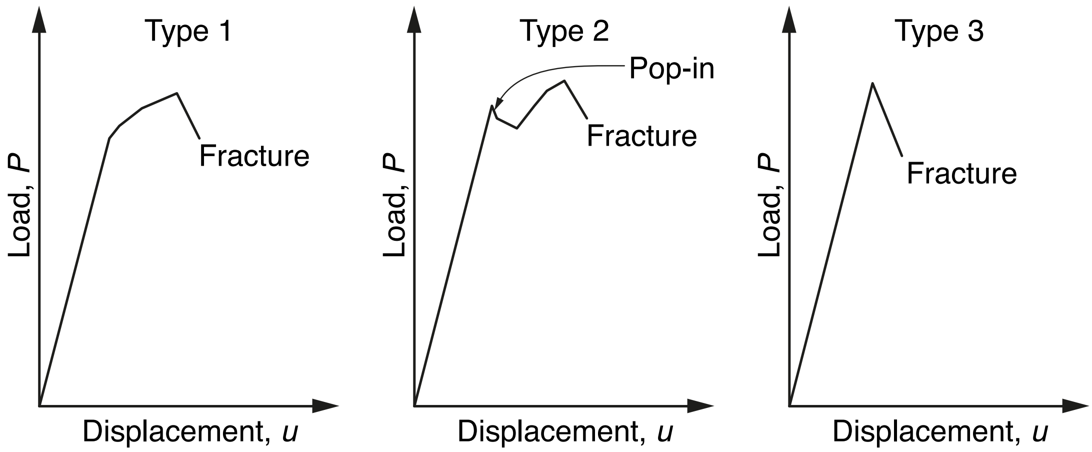
As previously noted it is a great advantage in trying to carry out a fracture toughness test to have some reasonable idea of the likely result since it is the result which effectively determines whether or not the procedure will have to be repeated.

1. Determine the critical dimensions of the specimen in terms of the required, a, $W - a$ and B to ensure true plane strain conditions. This can only be achieved by having a good idea of the probable fracture toughness or by 'guessing' the likely value of K_Q since it is this value which will determine the physical dimensions required of the specimen to obtain valid plane strain conditions at the crack tip. Quite often a Charpy or Izod toughness value is available and these values can be used in conjunction with empirically developed relations to predict a stress intensity fracture toughness.
2. Select an appropriate specimen geometry consistent with available material, testing machine and crack orientation.
3. Machine specimen to suit chosen geometry and fatigue pre-crack ensuring that fatigue pre-cracking load does not lead to a large crack tip plastic zone. Maximum applied K in the final stages of fatigue pre-cracking must be less than 60% of subsequently determined K_Q.
4. Set up specimen in testing machine with appropriate instrumentation to be able to record the load-displacement diagram with sufficient accuracy.
5. Test specimen recording a continuous graph of load versus displacement up to eventual fracture of the specimen.
6. Analyse load-displacement record.
7. Calculate K_Q on the basis of the chosen protocol method.
8. Check for validity of plane strain conditions and if satisfied then $K_Q = K_{Ic}$.

5.4 Analysis of load-displacement records

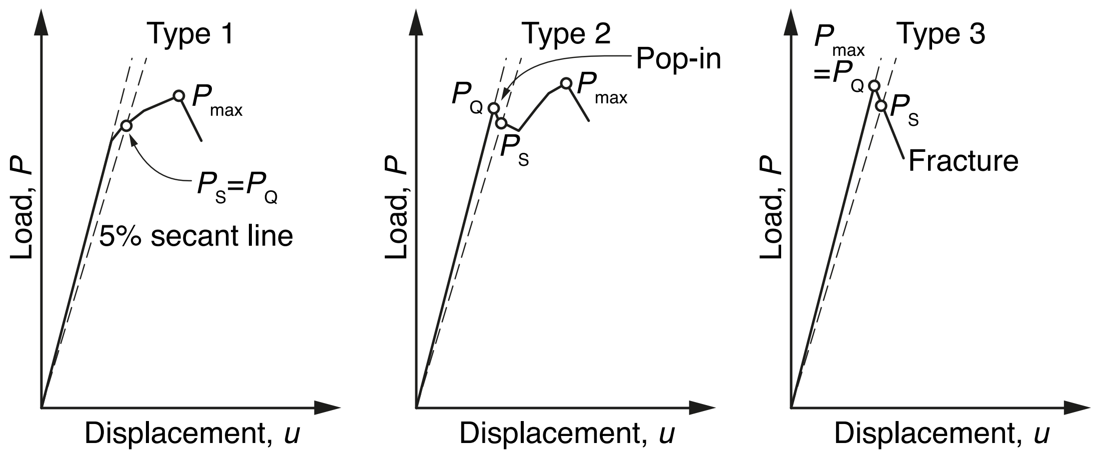
A systematic approach for classifying and analysing the widely varied load-displacement records from fracture tests.

Point 5 requires that a continuous record of load versus displacement should be made for each attempted plane strain fracture toughness test. In practice many different load-displacement records may result but invariably these can be categorised as belonging to one of the three following types:



The type 1 record is typical of most metallic materials. The non-linearity in the load-displacement record is due to increasing plasticity and/or stable crack growth at the crack tip prior to fast fracture. Sometimes a sudden burst of crack growth occurs at the crack tip which subsequently arrests and this phenomenon of a so-called 'pop-in' gives a load-displacement record typical of type 2. Very occasionally a type 3 record is encountered for materials which are classically linear elastic in their response to loading. Materials such as ceramics, grey iron or glass can exhibit this type of behaviour.

Having tested a specimen it is first necessary to decide which of the three load-displacement records best typifies the load-displacement record obtained. The load value at fracture to be used in the calculation of K_Q is then determined by graphical construction for load-displacement records of types 1, 2 or 3.



For example if the load-displacement record is of type 1 or 2 then the appropriate load for use in the stress intensity factor expression is found by constructing a secant line with a slope 5% less than the elastic loading line. The intersection of this secant line with the loading line is at *P*S. If the load at all points on the diagramprior to P_S is less than P_S then $P_S = P_Q$, as for example in the type 1 record. However if a load point on the diagram prior to P_S is greater than P_S then the appropriate load for calculation of K_Q is the maximum load preceding P_S, as for example in type 2.

There is a additional, necessary restriction in that for both types 1 and 2 the maximum load $\frac{P_{max}}{P_Q} \leq 1.10$ this restriction being intended to ensure that too much stable crack growth has not occurred prior to fracture.

If $\frac{P_{max}}{P_Q} \geq 1.10$ the test must either be repeated with a thicker specimen or the use of a plane strain fracture toughness protocol is inappropriate in which case a K_{Ic} approach to interpretation of the record should be adopted or an alternative measure of fracture toughness should be adopted. Where the load-displacement record conforms to type 3 then the maximum load recorded is the appropriate value to be used in calculating the provisional fracture toughness values.

Further restrictions apply to the shape of the crack front at the onset of brittle fracture which must be reasonably straight fronted to conform to the standard and if these restrictions are satisfied then the deduced value of stress intensity factor at fracture, K_Q is tested to ensure plane strain conditions.

If the dimensions of the specimen tested were such that:

$$B, W - a, a \geq 2.5\left(\frac{K_Q}{\sigma_y}\right)^2$$

then $K_Q = K_{Ic}$ and the value of stress intensity at brittle fracture is the plane strain fracture toughness which can be regarded as a material property in the same way as yield stress or tensile strength. In practice at least two specimens of the same geometry would be tested and the results should be within 5 to 10% for a satisfactory result to be recorded.

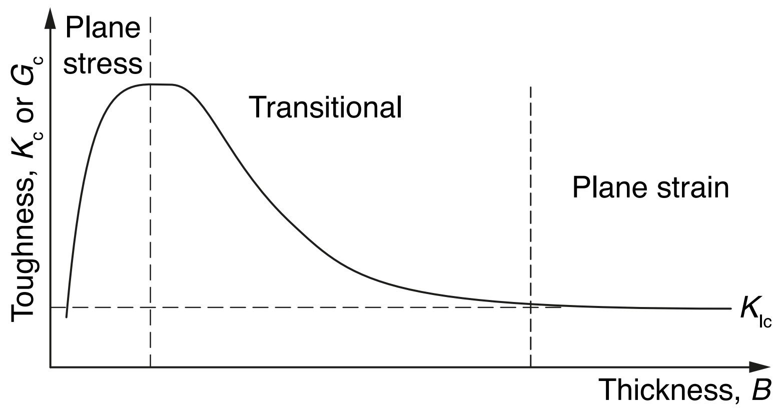
5.5 Plane stress fracture toughness (K_c) testing

Methods for testing materials when plane strain conditions are inappropriate or practically unattainable.

5.5.1 Plane stress region

In this region, toughness increases rapidly with thickness.

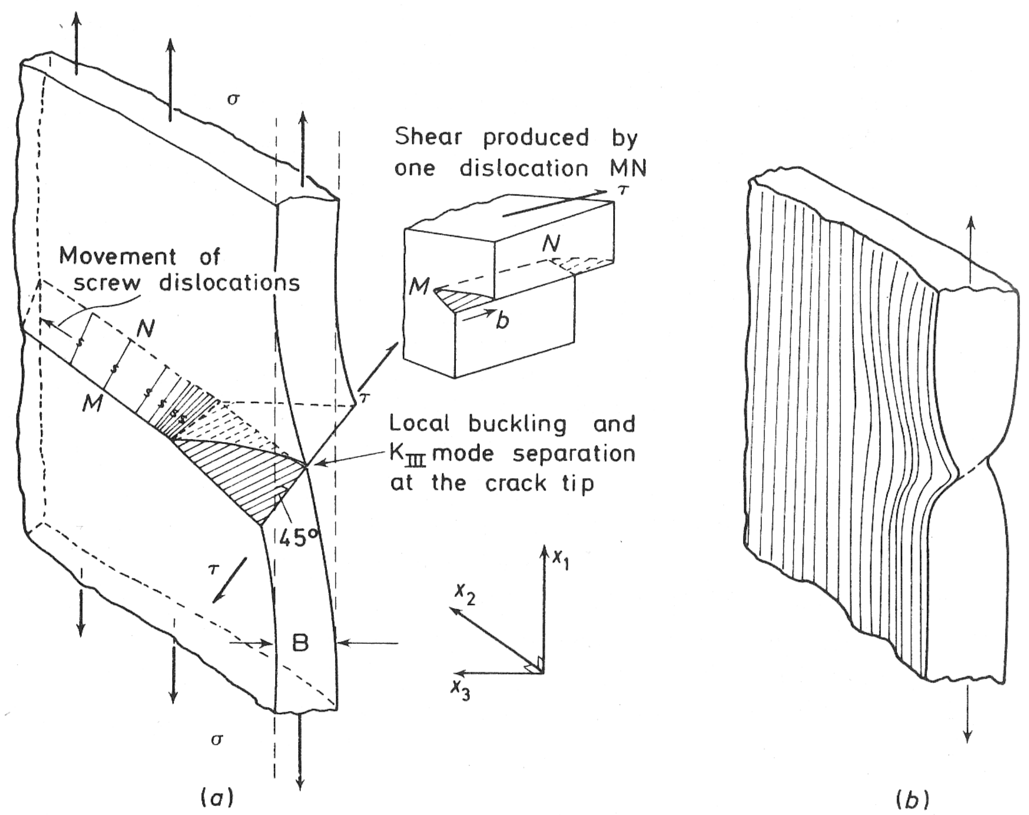
In order to describe an appropriate method for non-plane strain fracture toughness testing reference must again be made to the general toughness versus thickness relationship for metals.



Consider first the plane stress region in which we know from experience that the fracture occurs in a slant plane oriented at proximately 45° to the tensile axis.

An analysis due to Bilby *et al*. has shown that the anti-plane strain mode III deformation which occurs during the fracture of thin sheet can be described by considering the shear displacements which take place in the formation of a neck in the plate thickness prior to fracture. It is assumed that the criterion for crack extension is that the total shear displacements must equal the crack path which in this case of the 45° shear plane is $\sqrt{2}B$.

From JF Knott-Fundamentals of Fracture Mechanics:



Thus it can be shown that for a constant load situation the fracture stress is approximated by

$$\sigma = \sqrt{\frac{2\sqrt{2}E\sigma_y B}{\pi a(1+\nu)}} \ldots\ldots\ldots \text{(Eq. 2)}$$

and by substituting the Griffith expression for the plane stress failure stress we get that

$$G_c = \frac{2\sqrt{2}\sigma_y B}{(1+\nu)} \approx 2\sigma_y B \ldots\ldots\ldots \text{(Eq. 3)}$$

Thus this model predicts that the fracture toughness should be proportional to the thickness which is more or less what we observe experimentally at the lower end of the plane stress region with fracture toughness rising steeply with thickness. Hence it can be seen that for very thin sheet materials such as aluminium foil the fracture energy can be very low despite the facts that aluminium is nominally a very ductile, tough material and that when loaded under plane stress conditions we would normally regard this as being the optimum loading conditions for good toughness.

This type of plane stress fracture behaviour has practical implications in that inspection of Eq. (3) indicates that, contrary to all classic shibboleths of fracture, increased fracture toughness is obtained by increasing yield stress and thickness simultaneously. Hence in determining the membrane thickness for a thin container (e.g. Coca-Cola can?) it may well be a minimum fracture toughness which is the dominant criterion rather than a thickness determined from pressure carrying capacity.

The foregoing model of the fracture process may well be incorrect in detail but provides a satisfactory model for observed failure stresses and loads.

5.5.2 Transitional region

In this region the fracture toughness of a specimen or component is not dominated by one or other of the extreme effects with their associated fracture modes.

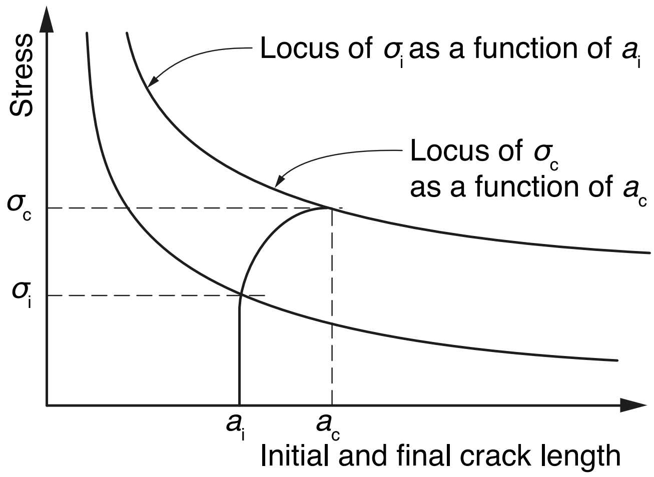
That is the fracture is neither wholly slant fracture indicative of plane stress and shear displacement control nor is it wholly square fracture indicative of plane strain conditions and a microvoid coalescence fracture mechanism. Indeed during the testing of a specimen of intermediate thickness the fracture mode changes as crack growth occurs. Initially a square pop-in fracture may occur tunnelling into the thickness but as the load is increased and further stable crack growth occurs the plastic zone size increases allowing the through-thickness stress to relax and thereby causing a reversion to shear controlled slant fracture.

The apparent complexity of the fracture process under these mixed mode conditions means that it is extremely doubtful whether a single instability criterion can be determined which will simultaneously take account of an increasing plastic zone size in competition with an increasing strain energy release rate.

5.5.3 *R*-curves

One approach to the measurement of fracture toughness in the transitional region.

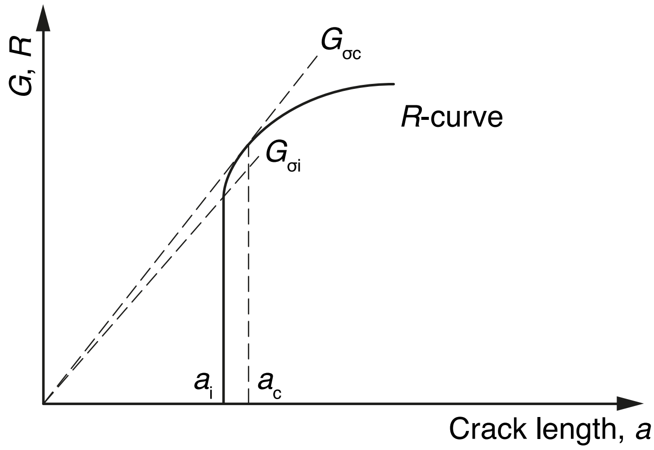
Imagine testing a series of thin (plane stress conditions) centre-cracked panels to instability with different initial crack lengths. The results of such test would appear schematically as shown below.



For an initial crack length a_i the crack begins to extend in a stable manner at a stress σ_i and crack length will not increase unless the applied stress is increased above σ_i. If the stress is increased then stable crack growth occurs up to some applied stress σ_c where instability intervenes and unstable crack growth results in fracture of the specimen. From the graph it is evident that the instability stress and crack lengths vary according to the initial crack lengths in the centre cracked panel. Under predominately plane stress conditions instability is almost always preceded by some slow growth.

The idea of an *R* or **resistance** curve is defined quite simply by saying that the resistance to crack extension, *R*, is equal to the rate of increase of surface energy for an infinitesimal increment of crack extension. Thus $R = 2\gamma$ which in turn equals *G* the strain energy release rate. For ideally brittle materials fracture occurs at a critical value of G_c and hence *R* is a constant for this class of materials.

For materials which can exhibit extensive plastic deformation prior to fracture the 2γ term is dominated by the plastic work to fracture and *R* is no longer sensibly constant.



Hence for the transition region a plot of R or G versus crack length will appear as a rising R-curve.

It can be shown that in plane stress or mixed mode conditions two conditions are necessarily satisfied before instability will occur:

1. $G = R$, and
2. $\frac{\partial G}{\partial a} = \frac{\partial R}{\partial a}$.

Whilst this is a useful concept in explaining the difficulties involved in determining fracture toughness in the transition region it is not very helpful in practice since scalable quantities are not easy to determine from practical R-curve tests.

In reality most practical cracked body situations cannot be easily modelled by carrying out an R-curve test and therefore most cracked body problems are simulated by carrying out a K_c test. Unfortunately there is no standard methodology available for the interpretation of K_c tests and the amount of slow crack growth occurring during such a test cannot be easily accomodated in the interpretation of the test record.

5.5.4 Plane stress K_c testing

The basic idea in practical K_c testing is that a value of stress intensity can be found which adequately describes the residual strength of a cracked body with a given initial crack length.

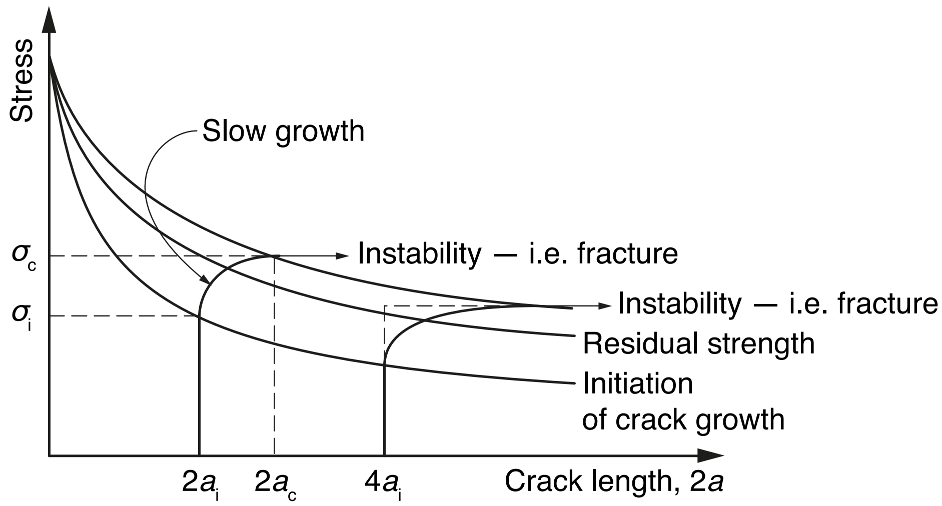
In carrying out the actual fracture tests necessary to construct a residual strength diagram several practical points should be observed:

1. It is important that the specimen thickness should be representative of the application.
2. A test geometry should be chosen for which the stress intensity factor expression is well documented and the chosen geometry should be one in which bending or buckling (in thin sections) is minimised.

3. Fatigue pre-cracking of the initial crack is advantageous but may not be necessary given that slow crack growth before fracture will often occur and cannot usually be avoided.
4. The maximum load in the test is taken as the P_Q value in calculating the stress intensity at fracture but it should be remembered that this instability stress intensity value is always above the residual strength value.
5. The amount of slow crack growth during the test should be monitored and recorded by whatever means are found to be most applicable.

A typical approach to residual strength determination is that due to Feddersen. The details of the derivation of this approach to plane stress fracture toughness testing are beyond the scope of this course but in general terms the approach can be described as follows.

If a series of centre cracked plate specimens with differing initial crack lengths are loaded in tension and tested to failure the results of these tests can be shown graphically:



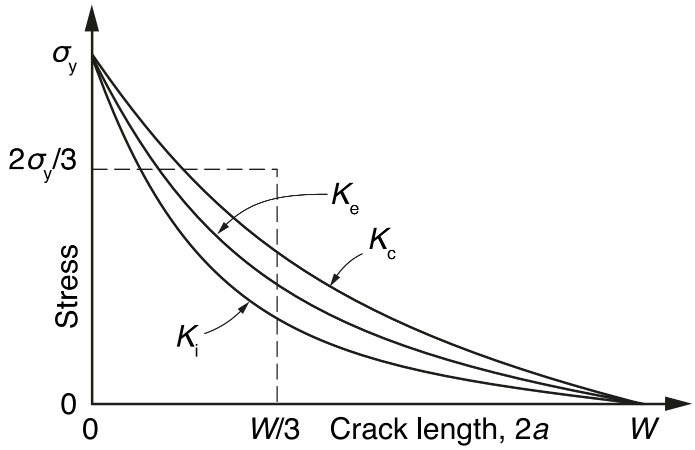
Values of stress can be determined for the onset of stable crack growth and instability for various initial crack lengths. A diagram is then constructed onto which these values are plotted. The residual strength line as a function of initial crack size is found by extrapolating back from instability stress and upwards from the initiation stress. The locus of the intersections is deemed to define the residual strength as a function of initial crack length. This approach can be justified since each event in the crack propagation and fracture is assumed to be controlled by the attainment of a certain value of stress intensity.

Thus the onset of stable crack growth is designated as K_i and the instability stress intensity is designated K_c. A further stress intensity value K_e is defined which is found to be representative of the residual strength of the plate with a given initial crack size. These stress intensity values are defined as follows

$$K_i = \sigma_i \sqrt{\pi a_i} f\left(\frac{a}{W}\right)$$

$$K_c = \sigma_c\sqrt{\pi a_c}f\left(\frac{a}{W}\right)$$

$$K_e = \sigma_c\sqrt{\pi a_i}f\left(\frac{a}{W}\right)$$



Although tests show that these terms are not true material properties in the sense of a K_{Ic} value the K_e value has engineering significance since when the data is replotted in terms of the material yield stress and normalised crack length the fracture toughness of the cracked plates is found to be well represented by the residual strength predicted by this approach.

Chapter 6 Crack opening displacement (COD)

A method of characterising failures which can be classified as ‘fracture’ but are too ductile for LEFM.

The previous parts of this course discussed methods and applications for predicting the onset of fracture in situations where the risk of crack propagation without appreciable amounts of plastic behaviour exists. In this situation engineering brittle fracture occurs (“breaks before it bends”) and this is a phenomenon that can be accurately described using LEFM. The fracture criterion developed for the brittle fracture situation requires that:

1. A material property (K_{Ic}) can measured for the material in question;
2. The applied stress intensity severity on the cracked component can be calculated from a knowledge of the shape factors plus load and crack length; and
3. The likelihood of fracture can be assessed by comparing the two.

Two situations may arise in which the use of the LEFM parameters becomes impossible.

1. Problems in measuring the material property. For some materials, the minimum thickness necessary to measure a valid K_{Ic} can make the test impossible to perform either through difficulties with the size of rig required, or due to material being unavailable in the required thickness range.
2. Problems arising from the applied stress intensity on the cracked component. There are situations in which the amount of plasticity near the crack tip invalidates the use of the LEFM approach to describe the applied severity.

This section describes a method of characterising failures that can occur by cracking with considerably less overall ductility than expected from a conventional tensile test, but with yielding extensive enough to make LEFM inapplicable. These situations arise typically for structural steels and aluminium alloys of moderate strength, and define the intended scope of Elastic Plastic Fracture Mechanics (EPFM).

The formal scheme for EPFM follows that of LEFM, in that an attempt is made to define a single parameter that measures the intensity of the deformations applied to a sharp crack. This time, however, the presence of plasticity is acknowledged rather than neglected. The material response is measured notionally by some critical value of that chosen parameter. So far, two such parameters have emerged:

1. The so-called J-contour integral; and
2. The critical value of crack opening displacement, COD.

The following sections describe approaches used to tackle the problem of fracture toughness testing in the presence of plasticity. Initially the plastic zone size correction is discussed as the first attempt to extend LEFM considerations and include plasticity effects. Then the crack opening displacement method is described in detail and the idea of a fracture assessment diagram (FAD) is introduced in very general terms.

6.1 The plastic zone correction

When plasticity which is restricted to a zone near the crack tip, a small adjustment to LEFM is sufficient.

The problem of determining the size or extent of the plastic zone ahead of a crack is a very complex one. This is due not only to the non-linearity of the plasticity constitutive equations, but also to the incremental nature of plasticity. Instead of attempting to completely describe the plastic zone in terms of shape and extent, a useful simplification is to assume a shape and, given that shape, to calculate the extent (i.e. a plastic zone size).

As a first attempt to account for plasticity we consider that a small plastic zone exists ahead of the crack tip and that the global effect of such a zone is to make the cracked body more compliant (less stiff). This is equivalent to the effect of having a longer crack. A tentative description of the effect is then made by using a virtual, longer crack length. The plastic zone is roughly quantified by considering the point where $\sigma_{yy} = \sigma_{\mathrm{y}}$. The value of r for which that happens is taken as the radius r_{p} of a small plastic zone. From the LEFM stress solutions this corresponds to:

$$r = \frac{1}{2\pi}\left(\frac{K^2}{\sigma_{\mathrm{y}}^2}\right) = r_{\mathrm{p}} \ldots\ldots\ldots \text{(Eq. 1)}$$

and the existing elastic analysis is retained for an effective crack of $2(a + r_{\mathrm{p}})$ instead of $2a$.

Since the whole solution is for elastic situations, only small plastic zones are acceptable and this is good enough for a first order correction factor. The final result is a measured G or K which is higher than the pure LEFM one, in agreement with the increase in fracture toughness expected due to plastic behaviour.

However it should be self evident that for larger amounts of yielding the argument does not hold.

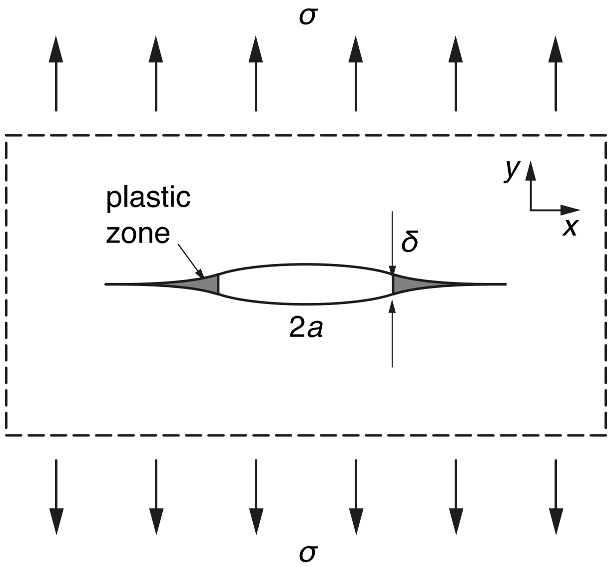
6.2 Crack opening displacement

COD can be used to characterise fracture where plastic flow is no longer small scale, but is restricted to the crack plane.

The concept of crack opening displacement, COD, was introduced in the UK by Wells in 1961. Wells argued that, due to the plastic response of a material, the stresses at the tip of a crack always attain a critical value at fracture and therefore, it is the plastic strain at the crack tip that controls fracture. Defining the crack opening displacement, COD, as the displacement between the faces of the plastically blunted crack (or crack flanks), it was postulated — and later numerically demonstrated by McMeeking — that the critical strain at the tip of the crack is a function of the COD. The onset of crack extension should then take place at some critical value of the COD.

The COD approach is more widely developed and accepted in the United Kingdom, as opposed to the trend found in the United States, where solutions for EPFM problems have been sought using the *J*-contour integral approach.

An analytical expression for the COD was first proposed by Burdekin and Stone and added strength to Wells' idea. They made use of the Dugdale strip yield model:



Analysing the Dugdale model and defining the COD as the displacement at the end of the strip yield zone yields the following expression for COD in an infinite plate with a central crack as a function of nominal stress applied, σ, and the crack length a:

$$\delta = \frac{8\sigma_y a}{\pi E} \ln\left[\sec\left(\frac{\pi\sigma}{2\sigma_y}\right)\right] \ldots\ldots\ldots \text{(Eq. 2)}$$

Based on this relationship, they also demonstrated that the intensity of the stress field ahead of the tip could be characterised by δ and that δ could be expressed as a function of the stress intensity factor K_I:

$$\delta = \frac{4}{\pi}\frac{K_1^2}{E\sigma_y} \approx \frac{K_1^2}{E\sigma_y} \ldots\ldots\ldots \text{(Eq. 3)}$$

assuming that $\sigma \ll \sigma_y$ σ σ_y and taking only the first term of the series expansion of the ln[sec()] part of the equation. Equation (3) above can also be written in terms of G as

$$\delta = \frac{G}{\sigma_y} \dots\dots\dots \text{(Eq. 4)}$$

within the assumptions of the strip yield model, namely for a state of plane stress and for a non-work hardening material. The equation can be rewritten to describe a wider group of materials as

$$\delta = \frac{G}{m\sigma_y} \dots\dots\dots \text{(Eq. 5)}$$

where m is a factor representative of a material for a particular level of constraint (thickness) and takes values between 1.0 for plane stress and about 2.0 for plane strain.

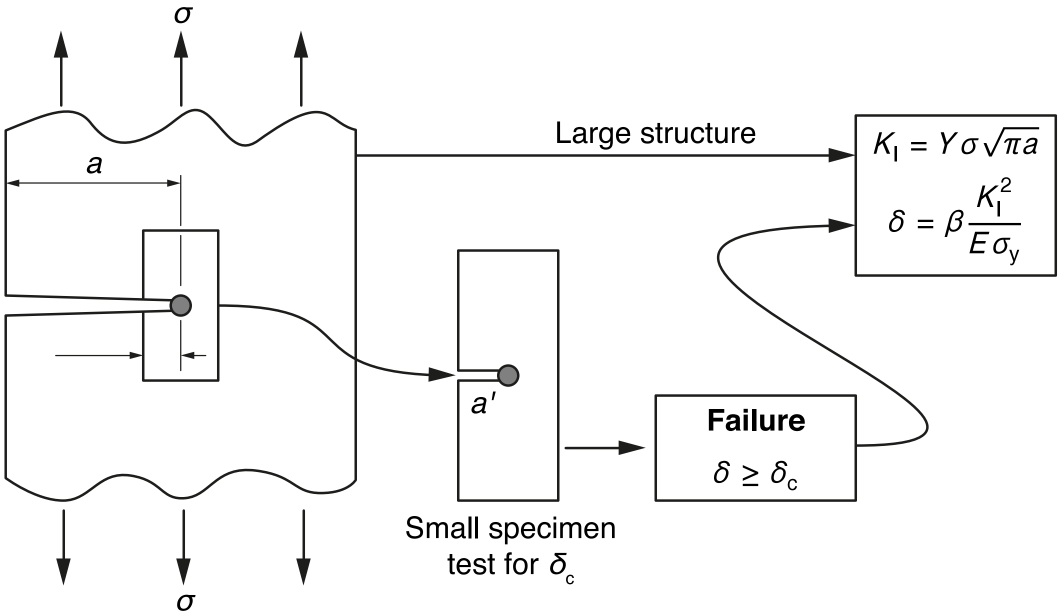
In spite of being restricted to the geometry for which it was developed — wide plate, centre crack — this relationship shows that, in the elastic regime, the COD approach is in accordance with the LEFM predictions. In addition to this, the COD approach is not intrinsically incompatible with situations beyond linear elastic limits, as LEFM is, because the occurrence of crack tip plasticity is part of the model itself.

When using COD in design it is usual to express the COD in a non-dimensional form φ:

$$\phi = \frac{\delta}{2\pi\epsilon_y a} \dots\dots\dots \text{(Eq. 6)},$$

where a is the crack length and ϵ_y is the strain at yield (as in σ_y/E). The use of the COD parameter in design is discussed in a later section. The underlying idea is that the critical value of COD is a fundamental quantity that can be transferred from the test piece on to the real structure and characterises failure even in the presence of plasticity.

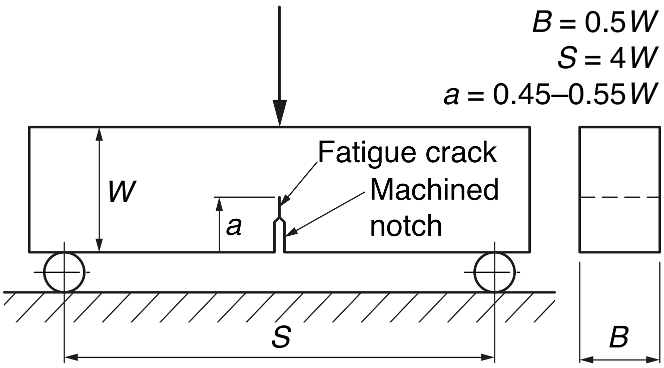
The basis of the COD approach to toughness characterisation is that the local processes resulting in separation at the crack tip are identical in the nominally elastic and yielding situations (provided full thickness of material is employed). If this is so, the critical COD, δ_c should be a characteristic of the material and the state of stress at the crack tip (i.e. δ_c is thickness dependent in the same way as K_{Ic}). Hence δ_c should characterise toughness in a similar manner to K_{Ic} or G_c.



6.3 COD testing

How to characterise a material by measuring the critical COD at which a crack begins to extend.

The COD toughness δ_c is measured in single edge notched bend (SENB) specimens so as to give conditions of high constraint at the crack tip. The thickness B of the specimen should be equal to the thickness used in service so that full through-thickness constraint is developed. Under these conditions any susceptibility to cleavage fracture will be seen in the test. The crack tip can be positioned in e.g. parent plate, HAZ or weld metal to test all potential fracture locations.

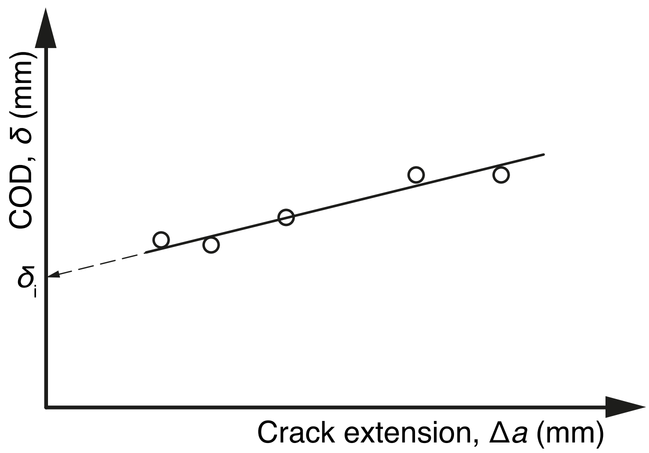


The opening displacement is measured at the mouth of the crack from which the displacement at the original crack tip, i.e. δ_c is calculated. Cracking may initiate at a lower value of COD, δ_i and develop in a stable manner before unstable propagation occurs at δ_c. Experiment shows that δ_i is too conservative for the definition of allowable crack sizes. Therefore δ_c measured at instability or maximum load is used as the basis for design against unstable fracture whereas the initiation value is used for standardised materials acceptance testing.

The standard COD testing method is defined in BS 7448-1:1991, which should be consulted for a complete description of the recommended testing procedure. Some of the requirements are listed below:

- Three point bend test piece recommended.
- With thickness B: $W = 2B$, $S = 4W$, $0.45 < a/W < 0.55$.
- Specimens should be fatigue precracked.
- Specimen mouth opening displacement should be monitored with a clip-gauge.

- **Method**
  - Perform several tests extending the crack by a certain different amount each time (all close to initiation). Calculate the COD for each one and plot against crack extension. Extrapolating backwards should give the COD for initiation δ_i:



  - COD is calculated via formulae given in the standard.

- **Comments**
  - COD testing can be used for tough, very ductile materials (structural steels, for instance) in which the extent of plasticity during a test would not enable the use of the LEFM testing standards.
  - Although *K/G* and δ can be related notionally as shown above, conversion from a measured δ to a G or K is **not** recommended practice in fracture mechanics design or assessment. The COD has the merit of allowing some plastic behaviour by definition and is used in design with the method of 'COD Design Curves' or through PD 6493.

6.4 COD design curves

These provide a first alternative to LEFM criteria for design.

The COD design curves were the basis of the first version of the Published Document 6493 (PD 6493:1980) in 1980. The underlying idea was a modification of the theoretical COD expression for the centre cracked plate to produce a lower bound design curve based on experimental data obtained from a large number of wide (1 m^2) welded steel panels. The validation testing was performed by the Welding Institute and the criterion proved extremely reliable.

Since COD is adopted as the describing parameter, the final design curve(s) should be applicable to real cracked structures with stresses approaching or in excess of the yield stress.

The method involves the determination of a material property (i.e. a critical COD value) and then, through use of the design curves, the critical COD determined from the laboratory situation can be used to predict the onset of fracture in a real structure.

6.5 COD design curves — stress basis

A much easier way to manipulate and calculate meaningful values for the fracture process in ductile materials.

For real crack geometries and for stresses approaching or in excess of yield stress, the theoretical centre-crack model is modified to produce a lower bound design curve which is based on experimental data obtained from a large number of wide (1m square) welded steel panels. There are two equations for predicting unstable fracture:

$$\bar{a}_{\mathrm{max}} = \frac{\delta_{\mathrm{c}} E \sigma_{\mathrm{y}}}{2\pi\sigma_1^2} \ldots\ldots\ldots \text{(Eq. 7)}$$

for $\frac{\sigma_1}{\sigma_{\mathrm{y}}} < 0.5$ and

and

$$\bar{a}_{\mathrm{max}} = \frac{\delta_{\mathrm{c}} E}{2\pi(\sigma_1 - 0.25\sigma_{\mathrm{y}})} \ldots\ldots\ldots \text{(Eq. 8)}$$

for $\frac{\sigma_1}{\sigma_y} > 0.5$.

$\bar{a}_{\mathrm{max}}$ is the maximum **tolerable** ‘effective’ crack size of related to the centre crack in a wide plate. In other words first we calculate the maximum allowable crack length for a centre-cracked plate and then translate this to our actual geometry and crack length through use of the LEFM shape factors, using the relationship

$$Y^2 a_{\mathrm{actual}} = \bar{a} \ldots\ldots\ldots \text{(Eq. 9)}$$

σ_1 is related to the nominal applied stress σ as follows:

Location of crack	Weld condition	Magnitude of σ_1
Remote from stress	Stress relieved	σ
	As-welded	$\sigma + \sigma_y$
At site of stress concentration SCF = k_t	Stress relieved	$k_t\sigma$
	As-welded	$k_t\sigma + \sigma_y$

Note

It is possible that $\sigma_1 > \sigma_y$ even for plane stress situations. This arises because the COD approach should properly be formulated in terms of strain rather than stress, i.e. σ_1 and σ_y should be replaced by ϵ_y and ϵ_y in the above equations. There is no problem in having ϵ_y greater ϵ_y. Usually it is more convenient to use the equations in terms of the equivalent notionally elastic stresses.

The design curve is intended to be deliberately **conservative**. Calculated tolerable defect sizes are typically a factor of 2 smaller than the actual **critical** defect size. This arises because the use of the Dugdale model assumes plane stress conditions which means that yielding is assumed to occur when the equivalent stress reaches σ_y. In reality most structures will require a higher equivalent stress to be achieved before yielding can occur, because of constraint. Thus the actual COD will be less than predicted by the design curve with a commensurately smaller tolerable crack size being predicted.

6.6 COD design curves — strain basis

The origin of the correlation between mouth opening and fracture toughness was based originally on strains.

As noted above the original and fundamental derivation of COD was formulated in terms of strains. For the sake of completeness it is worth running through the same procedures as in §6.6, but using the non- dimensionalised form of the COD :

$$\phi = \frac{\delta_c}{2\pi\epsilon_y\bar{a}} \dots\dots\dots \text{(Eq. 10)}$$

where ϵ_y is the strain at yield and $\bar{a}$ is related to the real crack length by

Important

$\bar{a} = Y^2 a$ (when $K = Y\sigma\sqrt{\pi a}$) ——— (Eq. 9 again)

In other words an effective crack length is determined from Eq. (9) through the use of the normal geometry factors which would be appropriate for the crack geometry and length in the body in the LEFM situation.

The non-dimensional COD is then related to nominal strain, ε, in the absence of the crack by

$$\phi = \left(\frac{\epsilon}{\epsilon_y}\right)^2 \text{ for } \frac{\epsilon}{\epsilon_y} \leq 0.5$$

and

$$\phi = \left(\frac{\epsilon}{\epsilon_y}\right) - 0.25 \text{ for } \frac{\epsilon}{\epsilon_y} \geq 0.5 \ldots\ldots\ldots \text{(Eq. 11).}$$

- The value of ϵ/ϵ_y is estimated from nominal elastic theory in the absence of a crack, i.e. -
  - for no stress concentration, $\epsilon = \sigma/E$;
  - for SCF = k_t, $\epsilon = k_t\sigma/E$;

  if the value of ϵ/ϵ_y exceeds 2, then a more accurate estimate of the strains is necessary (e.g. by finite element computation).
- If residual stresses exist, as for example in non-stress relieved weldments, the residual stress term, σr , is added if known or must assumed to be equal to the yield stress. In which case
  - $\epsilon = \frac{1}{E}(\sigma_r + K_t\sigma)$.

Thus for a given problem the strain ϵ in the structure is estimated and φ evaluated. Either the required material toughness or the critical crack size can be calculated from the relationships above in precisely the same manner as for the stress based method.

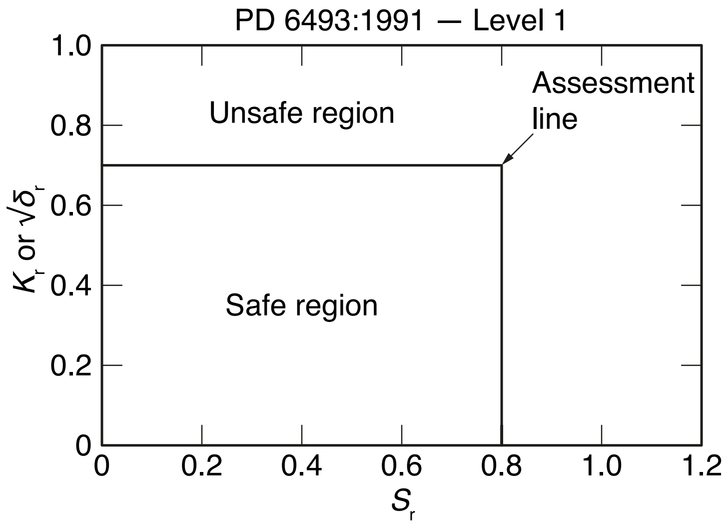
As previously stated the COD design curve is intended to be deliberately conservative. Calculated tolerable defect sizes are typically a factor of 2 smaller than the critical defect size. This conservatism is inherent in the formulation of the design curves from experimental data. However the COD method is arguably the best method for characterising weld metal toughness and several internationally accepted welding standards specify COD testing for quality control in the production of welded structures. In particular the production of the large welded jackets for North Sea oil rigs and the associated welded steel pipelines are all controlled through the specification of a minimum value of δ_c to be attained in the HAZ, weld metal and parent metal.

6.7 The Failure Assessment Diagram concept

For assessment of a given structure under extreme loading conditions, a FAD helps determine whether EPFM or plastic collapse is the dominant mechanism.

The boundary between EPFM and general yielding is subtle and loosely defined. In practice this means that structures loaded close to generalised yield loads should have a fracture assessment performed in conjunction with a plastic collapse assessment.

The concept of the failure assessment diagram (FAD) is so far the best solution to this problem. A failure assessment diagram is a two-dimensional diagram on which one region is defined as safe (and the remainder as unsafe) by a so-called assessment line. One of the axes of the diagram contains the fracture assessment and the other, the plastic collapse assessment. A particular structure under certain loading conditions is analysed using fracture parameters and general yield parameters. A point is then plotted on the diagram and a simple visual inspection assesses the conditions in question as safe or unsafe:



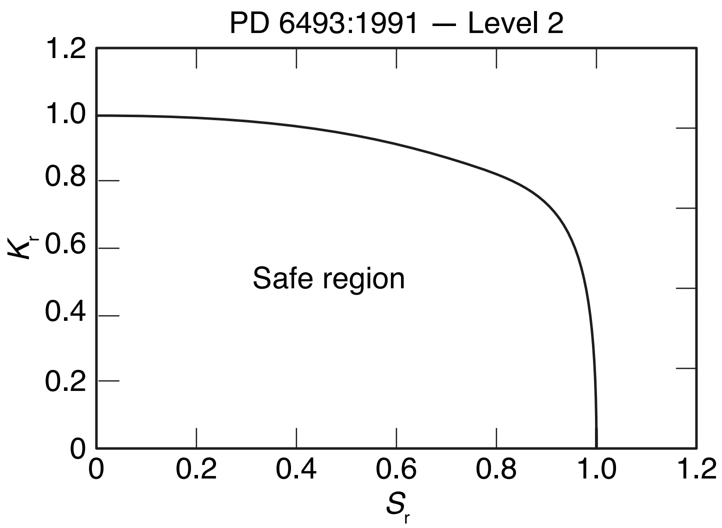
The PD document described in this section evolved to become a FAD-type method. In its first version (1980) PD 6493 proposed the fracture assessment in terms of the COD design curves. It consisted of a **design code**, in the sense that not only was a fracture assessment line proposed (i.e, the relationships themselves), but a whole set of assessment guidelines was given. The 1980 version included assessment against fracture, fatigue, overload, leakage, creep, buckling and environment effects.

In September 1999, a new version of the PD 6493 document was issued. The new version — called BS 7910 — has moved entirely towards the FAD concept.

6.8 Assessment methodology overview

BS PD 6493 sets out a general approach for assessing the stability of a cracked structure.

The FAD type of approach offers the advantage of immediate visualisation of the assessment cases. The latest document offers three levels of assessment which are used in accordance with the reliability of the data at the user's disposal as well as the time and effort available for the assessment task. Level 1 is the simplest and has an in-built safety factor. Level 2 is an intermediate option and the safety factor option is left entirely to the user. Level 3 is the most advanced form of assessment, it requires accurate and complete data both on the material side as well as on the loading condition on the structure.



In all three levels, the diagrams consist of a vertical axis where the fracture assessment is plotted and a horizontal axis for the plastic collapse assessment. This scheme allows the criteria for fracture and for plastic failure to be taken into account simultaneously via use of the assessment line. Rather than solving a coupled problem, this scheme allows the user to evaluate plasticity and fracture separately and let the diagram take care of combining the two. The two assessment axes are explained in the paragraphs that follow.

- **BS 7910:1999 - The Assessment Axes**

  The fracture assessment in the PD 6493:1991 provides FADs for the three possible assessment levels. In all three levels, the diagrams consist of a vertical axis where the fracture assessment parameter is plotted and a horizontal axis for the plastic collapse assessment parameter. Although subtle differences exist in the way the two assessment parameters are calculated from one level to the other, the axes are broadly as described in the following paragraphs.

- **The plastic collapse axis (horizontal axis)**

  The parameter plotted on the horizontal axis is denominated S_r for levels 1 and 2 and L_r for level 3 and is defined as the ratio of the effective net section stress on the structure to the material flow stress. Alternatively, this parameter can be defined as the ratio of the applied load over the plastic collapse load. Therefore for evaluation of the S_r parameter, it is necessary to obtain an expression for (or to calculate numerically) the effective net section stress on the cracked structure in question. Several references exist for such expressions. There are 21 Annexes to BS7910 which contain a short list of some usual expressions for effective net section stresses.

  The main general recommendations on the determination of the S_r parameter are:

  - Only primary stresses (i.e. those related with the external loads applied to the component) are included in the S_r calculation. Residual stresses or stresses from local stress concentrations are not included in the calculation, since they cannot cause plastic collapse. Due to being denominated as primary stresses or loads, the symbol 'p' is sometimes used to describe such stresses or loads.

Note

The terminology for stresses adopted in BS 7910:1999 is based on ASME VIII.

- Evaluation of S_r is usually based upon the value of the flow stress, defined arbitrarily as the average between the yield stress and the ultimate tensile stress (up to a limit of $1.2\sigma_y$). This value is regarded as more meaningful than that of the yield stress, since it implies some allowance for work hardening.

The term 'collapse load' as adopted in the document does not correspond to the strict definition of collapse for a structure as a whole, but more as a rather localised event, where yield spreads through the wall thickness, e.g. for a part-through thickness flaw. For that reason, the term 'effective net section stress' is preferred.

- **The fracture axis (vertical axis)**

For the fracture assessment, the applied stress system is again subdivided into primary stresses (those that can cause collapse) and secondary stresses (such as residual stresses). The fracture assessment parameter is denominated K_r and is defined as the ratio of the total applied stress intensity factor K over the material's failure K. Primary and secondary stresses are treated in a slightly different manner, basically due to the assumption that the plasticity associated with the primary stresses should be accounted for by the horizontal axis of the assessment diagram. In other words, plasticity from secondary stresses might cause fracture, even though it might not cause collapse so it must be fully allowed for in the fracture assessment.

The applied stress intensity factor K is evaluated just as in LEFM, using the Y shape factors. Unknown residual stresses are assumed to be equal to the value of the material's yield stress.

The material K_{Ic} value is used if known. It may not be available because of the difficulties in fracture testing for certain materials. In that case, a plasticity measure of toughness must be used — possibly a COD value or a K_{Ic} value obtained indirectly from a J-type test.

- **The Assessment Levels**

Level…

1. …is the simplest and has a large in-built safety factor.
2. …is an intermediate option where the safety factor is left entirely to the user.

3. …is the most advanced form of assessment, requiring accurate and complete data for both the material properties as well as the loading conditions on the structure.